Exercises

AM 0219: Nonlinear Dynamical Systems

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Exercise 9: Solve the following initial-value problems by separation of variables and determine the maximal time intervals of existence of the solutions:

(i) $\dot{x} = x^3 e^t, \qquad x(0) = 2,$

(ii) $\dot{x} = 4 + x^2, \qquad x(0) = 0,$

(iii) $\dot{x} = 1 - x^2, \qquad x(0) = 0.$

Exercise 10: Identify the ω-limits of <u>each</u> trajectory of

(i) $\dot{x} = x^3 + 3x^2 - 6x - 8, \qquad x \in \mathbb{R}$

(ii) $\dot{x} = \sin(x), \qquad x \in \mathbb{R}$

(iii) $\begin{pmatrix} \dot{x_1} \\ \dot{x_2} \end{pmatrix} = \begin{pmatrix} \cos x_2 \\ \cos x_1 \end{pmatrix}, \qquad x = (x_1, x_2) \in \mathbb{R}^2$

Exercise 11: A (point-sized) dog catches a bone at the origin $(x, y) = (0, 0)$ of the plane $\mathbb{R}^2$ and starts to run along the positive x-axis with speed 1. At the same moment a second dog starts at the point $(x, y) = (0, d)$ and chases the first one. The second dog has the same speed 1 and is always on target for the bone.

How close does the second dog get?

Hint: Use appropriate coordinates (e.g. r = distance of the dogs, φ = angle of the connecting line of the dogs with the x-axis) and solve the resulting system by separation of variables.

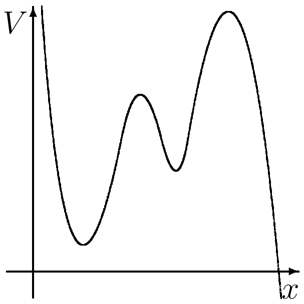
Exercise 12: Sketch the phase portraits of $\ddot{x} + V'(x) = 0$,

(i) for the Kepler problem,

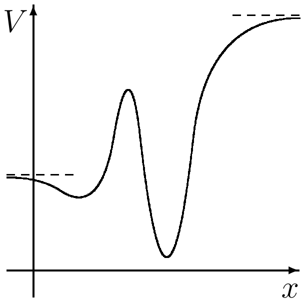
$$V(x) = -\frac{1}{x} + C\frac{1}{x^2},$$

$C > 0$, $x > 0$

(ii)



(iii)



Pay attention to saddle equilibria, homoclinic orbits, and asymptotic behavior at infinity.