Exercises

AM 0219: Nonlinear Dynamical Systems

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Exercise 21: Consider the Banach space BC^1 of continuously differentiable vector fields $f:\ X \to X = \mathbb{R}^n$ with

$$\|f\|_{BC^1} := \sup_{x\in X}(|f(x)| + |f'(x)|) < \infty.$$

Let f, g be vector fields in BC^1 and $x(f,t)$ denote the solution at time t of the differential equation

$$\dot{x}(t) = f(x(t)), \qquad x(0) = x_0.$$

Is the map

$$x(t,\cdot):\ BC^1 \to X, \qquad f \mapsto x(t,f),$$

differentiable with respect to $f \in BC^1$, for fixed t ? If so then which differential equation is solved by the variation $v(t) := \mathrm{D}_f x(t,f)g$?

Exercise 22: Find a counterexample to the following claim:

$$\mathrm{e}^A\mathrm{e}^B = \mathrm{e}^B\mathrm{e}^A,$$

for all real (2×2)-matrices A, B.

Exercise 23: Calculate the Picard iterates for the equation

$$\begin{aligned} \dot{x}(t) &= Ax(t), \qquad x\in\mathbb{R}^n,\ A\in\mathbb{R}^{n\times n}, \\ x(0) &= x_0, \end{aligned}$$

explicitly. The initial function is $x^0(t) \equiv x_0$. On which interval does the iteration converge?

Exercise 24: Let $A = (a_{ij})_{1\le i,j\le n}$ be a real $(n\times n)$-matrix. Prove: The coefficients of the matrix e^{At} are non-negative for all $t \ge 0$ if, and only if, $a_{ij} \ge 0$ for all $i \ne j$.

Hint: It suffices to consider the case $a_{ij} \ge 0$ for *all* i, j. (Why?)