Exercises

AM 0219: Nonlinear Dynamical Systems

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Exercise 33: How many digits (in the decimal system) does the 1.000.000.001-st entry of the sequence $(1, 3, 8, 20, 48, 112, \ldots)$ have, i.e. $x_n = 4x_{n-1} - 4x_{n-2}$ with $x_0 = 1$ and $x_1 = 3$?

Exercise 34: Let f be a vector field such that each trajectory is bounded.

Prove or disprove: The ω-limit depends continuously on the initial condition, i.e. if

$$\lim_{n\to\infty} \operatorname{dist}(x_n, x) \;=\; 0,$$

then

$$\lim_{n\to\infty} \operatorname{dist}(\omega(x_n), \omega(x)) \;=\; 0.$$

Here, the distance is defined as

$$\operatorname{dist}(A, B) \;:=\; \inf_{a\in A} \inf_{b\in B} \operatorname{dist}(a, b).$$

Exercise 35: Consider a continuous flow on X and a non-empty, compact, and invariant subset $M \subset X$.

Prove or disprove: M is stable if, and only if, every neighborhood of M contains a positively invariant neighborhood of M.

Hint: A neighborhood of a set A in Y is any set N which contains an open set U such that $\operatorname{clos}(A) \subseteq U \subseteq N \subseteq Y$.

Exercise 36: The theorem of GROBMAN&HARTMAN ensures the C^0 flow equivalence of vector fields to their linearizations near hyperbolic equilibria.

Find two (simple) examples of vector fields with a *non-hyperbolic* equilibrium, one which is C^0 flow equivalent to its linearization and one which is not.