Exercises

AM 0219: Nonlinear Dynamical Systems

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Exercise 41: Let $X \subset \mathbb{R}^2$ be a circular disc with ℓ disjoint circular holes and f a continuously differentiable vector field on X with $\operatorname{div} f > 0$.

Prove: the corresponding flow contains at most ℓ periodic orbits in X.

Exercise 42: Consider the differential equation

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= (1 - x^2 - y^2)y - x. \end{aligned}$$

Prove: there exists a *unique* periodic orbit.

Exercise 43: Let $x = 0$ be an *isolated* equilibrium of a flow f in $\mathbb{R}^2$. Prove:

(i) $x = 0$ is stable but not an attractor if, and only if, every neighborhood of $x = 0$ contains (at least) one periodic orbit (with positive minimal period).

(ii) If there exists a C^2 Lyapunov function V with $\nabla V(0) = 0$ and strictly indefinite Hessian matrix $\nabla^2 V(0)$ then $x = 0$ is unstable.

Exercise 44: Prove or disprove the theorem of Poincaré & Bendixson for flows on

(i) the sphere S^2,

(ii) the torus T^2.