Exercises

AM 0219: Nonlinear Dynamical Systems

Bernold Fiedler, Stefan Liebscher

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Exercise 45: Let f be a planar C^1 vector field and let the forward orbit $\gamma_+(x_0)$ of some initial condition $x_0 \in \mathbb{R}^2$ be bounded. Assume that $\omega(x_0)$ is neither an equilibrium nor a periodic orbit.

Prove: $\omega(x_0) = E \cup H$ is the union of

(i) a set E containing only equilibria and

(ii) a set H containing only homoclinic or heteroclinic orbits.

Can H contain countably/uncountably many orbits?

Exercise 46: Consider the prey-predator system

$$\begin{aligned} \dot{x} &= x(1 - ax - y), \\ \dot{y} &= y(-c + x - by), \end{aligned}$$

with $(x, y) \in \mathbb{R}^2_+$, and parameters $a > 0$, $b > 0$, $c > 0$, $ac < 1$.

Prove:

(i) there exists a unique equilibrium (x_*, y_*);

(ii) $\omega((x_0, y_0)) = \{(x_*, y_*)\}$, for all initial conditions $x_0 > 0$, $y_0 > 0$.

Hint: Remember the case $a = b = 0$, and compare with it.

Exercise 47: Choose two theorems on autonomous differential equations from our course. Formulate (reasonable) analogues for discrete-time dynamical systems. Decide whether your self-formulated analogues are true or false.

Exercise 48: [Floquet theory for discrete dynamical systems] Consider the iteration

$$x_{k+1} = A_k x_k$$

with $A_{k+p} = A_k$ for all $k \in \mathbb{N}$, and some fixed period $p \in \mathbb{N}$. Assume all matrices A_k to be invertible.

Prove: there exist matrices B, C_k, $k \in \mathbb{N}$ with $C_k = C_{k+p}$ and

$$\prod_{k=0}^{m} A_k = C_m B^m \qquad \text{for all} \quad m \in \mathbb{N}.$$