Evening Lecture and Concert

Shaping Musical Performance by Genealogical Trees of Vector Fields

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In contrast to the quasi-digital notation of music in European standard scores, performance involves infinitesimal deformations in time, pitch, durations, loudness, articulation and tuning. How can this shaping process–which Theodor W. Adorno termed "micrologic of infinite precision"–be controlled? Generalizing the well-known tempo curve, it is seen that special vector fields, the performance fields, adequately conceptualize Adorno's infinitesimals of micrologic. However, developing performance fields is not a one-time action, it must be built in a genealogical way from coarse to more and more refined performances. Performance grammars have to describe this continuous refinement process of shaping artistic performance. We describe and illustrate the genealogical stemma theory of performance, its implementation in the analysis and performance software RUBATO and present examples of performances which have been generated by such software on MIDI grand pianos.

A. Dynamical Systems

1.1 Celestial Mechanics

Organizer : Kenneth Meyer

Key note lecture

Horseshoes and the Problem of the Gaps

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A normally hyperbolic invariant annulus with transverse homoclinic points exists in the Newtonian five-body problem. In an attempt to understand the implications of this we show that there is actually an invariant Cantor set of annuli nearby. The dynamics on this set is a skew-product over a horseshoe map. Such a map can be viewed as a "random" map of the annulus. By making use of this randomness, one can avoid the problem of gaps between invariant circles on the original annulus which can be barriers to Arnold diffusion.

Invited lectures

Secular System in Souriau Variables

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The idea of using the "Laplace Vector" and the angular momentum vector as orbital elements comes from the fundamental paper of Lagrange "Théorie des variations séculaires des éléments des planètes" (1781).

I present here a work of my student K. Abdullah showing how a slight modification of this approach leads to the appearance of surprising properties of the secular part of the perturbating function.

Regularizable and non–regularizable collisions in n–body problems

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Collisions appear as singularities in the n–body problem. A binary collision occurs when only two masses collide. If all the masses coincide at the same point we get a total collision. Between these two cases one can get different types of collisions, for instance, triple collision (TC), simultaneous binary collisions (SBC), simultaneous binary and triple collisions, etc.

If a solution of Newton equations of motion is defined on some maximal interval $[0, t_c)$ with $t_c < \infty$, it is said that it has a singularity at $t = t_c$. Moroever if the masses approach a limiting position as $t \to t_c$, it can be proved that the limiting position must be a collision point. It is said that the solution ends at a collision.

The extension of a collision solution for $t \geq t_c$ has been studied in several cases. If the extension is possible in some sense it is said that the collision is regularizable in that sense. In that case one can also look for the smoothness of the regularization. If the collision is non–regularizable, it is interesting to describe which are the dynamical consequences. If small changes in the initial conditions lead to substantial changes in the solutions, how one can classify the possible transitions from approaching the collision to escaping from it?

We consider a geometrical approach, searching for continuity with respect to initial conditions for orbits passing close to a collision. After surveying known results, some recent progress, dealing with finite order of smoothness in the SBC case ([1],[2]) and with the classificacion of passages near TC ([3],[4]), is presented.

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On the homology of the integral manifolds in celestial mechanics

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The integral manifolds of the N-body problem are the level sets of energy, angular momemtum, linear momentum and center of mass. They are invariant under the flow, and their topology influences the possible dynamics of the system. Understanding their structure, and how that structure depends on the energy and angualar momentum parameters, is fundamental to understanding the global structure of the N-body problem. Recently, we have made progress in describing the topology of the integral manifolds, and detecting changes in that topology, by analyzing the homology of the manifolds. This talk will survey these results.

Melnikov Techniques in the Sitnikov Problem

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Using Melnikov techniques we get two different families of periodic orbits in the Sitnikov problem. The first family is obtained using a generalization of the Hartman -Grobman theorem that we prove here; and the second one is gotten using the clasical horseshoe of Smale. Both families are obtained analytically. These results improve a previous one of H. Dankowicz and P. Holmes.

Existence and Stability of Relative Equilibria in the n-Body Problem

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Special periodic solutions of the Newtonian n-body problem consisting of configurations of bodies rotating rigidly about their centers of mass are called *relative equilibria*. These solutions become fixed points in a rotating coordinate frame, hence the name. One important conjecture in the field, attributed to Moeckel, asserts that the only linearly stable relative equilibria are those containing a dominant mass, ie. one mass which is much larger than all the others. For example, placing n equal masses at the vertices of a regular n-gon with an additional body of mass m at the center yields a relative equilibrium for all values of m. If m is sufficiently large, then this configuration, referred to as the $1+n$-gon, is linearly stable.

We provide some results in support of Moeckel's dominant mass conjecture. One of these states that any relative equilibrium of n equal masses is not spectrally stable provided $n \geq 24,306$. We also consider a limiting problem with n "big" masses and p "small" masses, with the small masses being order ϵ [1]. This problem splits into two parts, where the n big bodies form a relative equilibrium of the n-body problem and the limiting positions ($\epsilon = 0$) of the p small bodies are critical points of a two-dimensional potential function. Assuming that the small masses do not coalesce in the limit, a case studied by Moeckel in [2], we derive sufficient conditions on the limiting positions of the bodies to insure that the entire family is linearly stable for ϵ sufficiently small. We then apply these conditions to the $1+n$-gon family showing that it is possible to thicken the ring around the central body and maintain its stability.

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The Stability of the Thomson Heptagon in an Ideal Fluid

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In his Adams prize essay of 1882 J. J. Thomson showed that in a uniformly rotating coordinate system a regular polygon configuration of N vortices is linearly stable for $N < 7$ and unstable for $N > 7$. The case of the heptagon can not be decided with the help of the linear terms. G. J. Mertz showed in 1978 with methods from fluid mechanics that the heptagon configuration is stable. By using methods common in celestial mechanics we can recover the result of Mertz and prove in addition that the heptagon configuration is locally Liapunov stable except for the rotational symmetry.

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Contributed talks

Barrier to transport in point vortex dynamics

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We study the dynamics of a system of identical point vortices and of test particles transported by the velocity field created by the vortices. Of particular interest is to determine how the specific vortex dynamics (periodic, quasi-periodic or chaotic) affect the transport and diffusion properties of the test particles; and the process of vortex cluster formation and possible trapping of particles at the interior of each vortex cluster.

For the case of four identical point vortices (*i.e.*, vortices with identical circulation) we have found that, by conservation laws, the vortex mouvement is cofined to a finite region of radius $R_v = \sqrt{L/k}$. The particle phase space is therefore partitioned into an inner region, where the vortex motion takes place, and outer region, from which the vortices are excluded.

We have found that in case of *quasi-periodic* vortex motion, there is a sharp barrier to transport between the central chaotic region, punctured with regular islands, and the outer region where the trajectories are on tori. Mixing and diffusion are therefore only possible in the central region which exhibits in this case a lobe structure, with several islands in addition to the ones encircling the vortices. The existence of such a sharp barrier to transport between regular and chaotic region is well understood by applying the *frequency analysis method* of Laskar[1], and corresponds to the existence of an invariant surface of quasi-periodic orbites.

Finally, for the subspace of symmetrical vortex configurations, the formation of vortex clusters can be explain by the existence of a bifurcation energy beyond which the system of four vortices becomes equivalent to a system of two or three vortex clusters.

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Positive solution II-nd part of 16-th Hilbert's problems

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1. Nonlocal (first) integral of polynomial vector field on the plane. Notations. Denote by $\mathbf{v}$ polinomial of degree n vector field on the plane; by P_j $(0 \leq j \leq n)$ different points on the plane; by t_j the time on the phase curve going through the point P_j, so $x(t_j)$ would be integral curve of vector field $\mathbf{v}$; by $f_j(x)$ $(0 \leq j \leq n)$ smooth function on phase plane, excluding probably some isolated phase curve of vector field $\mathbf{v}$. **Definition.**Iff there exist some diffeomorphisms $g_j(t)$ $(0 \leq j \leq n)$ such, that

$$H(P_j, t) = \sum_{j=1}^{n} f_j(x(t_j)) \,|_{t_j = g_j(t)} \equiv Const, \tag{1}$$

when we call the function $H(P_j, t)$ the (correspondingly, global) non-local first integral of vector field $\mathbf{v}$ (iff in (1) the time goes the whole of line $\mathbf{R}$). We call $x(g_j(t))$ $(0 \leq j \leq n)$ as support of non-local first integral.

Theorem. Let vector field $\mathbf{v}$ has convex periodic phase curve. Then in exterior of these circle there exist global non-local first integral of $\mathbf{v}$, iff there exist n circles of $\mathbf{v}$ in the above exterior.

2. Reduction to convex case. *Lemma.* Let us polynomial vector field $\mathbf{v}$ has k periodical integral curves. Then there exists one parameter family of vector fields $\mathbf{v}_\varepsilon$ with $deg\mathbf{v}_\varepsilon = deg\mathbf{v} + 4$ so, that $\mathbf{v}_0$ orbitally equivalent to $\mathbf{v}$ and for small value of parameter the vector field $\mathbf{v}_\varepsilon$ has at least k periodical integral curves. Moreover in its interior there is the smallest convex periodical integral curve.

3. Upper bound. For nesting periodical orbits of polynomial vector field the number of limit circles isn't greater than $n(n+1)/2$ for $n \geq 4$ (correspondingly, 6 for $n = 2, 9$ for $n = 3$).

Proof (see [1]) For above constructions it is needed $(n+1)$ circles. It is needed to remove critical values of global nonlocal integral by help of suitable number of limit circles ($n(n+1)/2$ for $n \geq 4$).

4. Bogdanov-Taken's bifurcation (see [2]) deals with normal form of degree 2. Appearing limit circles are convex. From above follows that it's number isn't greater than 6 (realy 1, see(2)).

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Periodic solutions of the spatial elliptic restricted three body problem

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We show the existence of periodic solutions in the three-dimensional elliptic restricted three body problem when the masses of the two primaries are equal. These solutions are perturbations of circular orbits of the Kepler problem when the massless body is far away from the primaries and its orbital plane is perpendicular to that of the primaries. The orbits are doubly symmetric and exist for a discrete sequence of values of the mean motion.

We scale the equations of motion so that the infinitesimal body is at distance unity from the primaries which are close to each other and move very fast. The distance between the primaries is taken as a small parameter but then the perturbation of the Kepler problem fails to be analytic. We then use a modified version of Poincaré's method of analytic continuation.

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$2n\pi$-Periodic Solutions of the Sitnikov Problem

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In this paper we give criteria for the detection and generation of $2n\pi$-periodic solutions of the Sitnikov problem. The Sitnikov problem is the simplest case of the spatial restricted problem of three bodies and is governed by

$$\ddot{z} + z/(z^2 + r(t)^2)^{3/2} = 0, \tag{1}$$

where $r(t)$ is a periodic function, $r(t) = r(t+T)$. In the normalized coordinate system we have $T = 2\pi$ and $2r(t) = (1 - e\cos E)$ with $0 \leq e < 1$ and

$$\cos E = -e/2 + \sum_{n=1}^{\infty}(1/n)[J_{n-1}(ne) - J_{n+1}(ne)]\cos nt. \tag{2}$$

This problem admits no classical integrals of motion. There are many attempts in dealing with this problem among which Moser's contribution has significant importance [1]. The existence of chaotic solutions near the escaping orbits were studied by Moser. Dankowicz and Holmes [2] reconsider the problem using Melnikov's theory and prove the existence of horseshoes. In our previous work [3], we dealt with Sitnikov's problem near the 3:2 resonance and studied its geometrical features when $|z| \ll |r(t)|$. In this study we present a method by which we can isolate $2n\pi$-periodic solutions. we use the following "integrable" equations

$$\ddot{x}_k + x_k/(x_k^2 + M_k^2)^{3/2} = 0, \quad k = 1, 2, \tag{3}$$

where M_1 and M_2 are the maximum and minimum values of $r(t)$ for $t \in [0, 2\pi[$, respectively. We show that at certain circumstances, the solution of (1) lies between x_1 and x_2. This allows us to express the evolution of the system in a "convergent" Hamiltonian of the form

$$\mathcal{H} = \frac{1}{2}\dot{u}^2 + \sum_{n=0}^{\infty}\Phi_n(t)u^n, \tag{4}$$

where Φ_n are 2π-periodic functions of t. The new dependent variable $u(t)$ is defined in terms of $x_1(t)$ and $r(t)$. Having $u(t) = u(t+2\pi)$ determined from (4), one could be able to construct $2n\pi$-periodic solutions of (1).

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Parabolic fixed points and stability criteria for nonlinear Hill's equation

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We discuss the stability of parabolic fixed points of area-preserving mappings and obtain a new proof of a criterion due to Simó. These results are employed to discuss the stability of the equilibrium of the periodic differential equation

$$\ddot{x} + a(t)x + c(t)x^{2n+1} + \ldots = 0 \; (n \geq 1),$$

when the variational equation at $x = 0$ is unstable-parabolic. In this class of equations the first Lyapunov's Method does not apply but in many cases the stability can be characterized in terms of the variational equation. We assume that $c(t)$ does not identically vanish and does not change sign. We associate two numbers $\sigma,\ \nu \in \{1, -1\}$ to the variational equation and prove that $x = 0$ is stable if $\sigma\nu c(t) \geq 0$ and unstable if $\sigma\nu c(t) \leq 0$. As a consequence, we obtain a necesary and sufficient condition for the stability of the equilibrium of the restricted three-body problem.

This is joint work with Rafael Ortega.

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Study of Complex Instability Using Normal Forms

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We consider a Hamiltonian H of three degrees of freedom and a family of periodic orbits with a transition from stability to complex instability, such that there is an irrational collision of the Floquet eigenvalues of opposite sign. We analyze the local dynamics around the transition studying the Hamiltonian from an analytical point of view using and *ad hoc* normal form. Carrying out a finite number of steps of this normal form process, and skipping the non integrable remainder, the existence of a bifurcating family of two-dimensional tori is derived. This family of tori may unfold on the unstable side of the family of periodic orbits, and then they turn out to be stable (*direct* Hamiltonian Hopf bifurcation), or they may appear on the stable side, being now unstable (*inverse* bifurcation). The existence of a direct or inverse bifurcation depends intrinsically on the Hamiltonian.

Our next objective is to show that, when the above reminder is added, most part (in the sense of the measure theory), of these 2D invariant tori persist in the complete Hamiltonian H.

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1.2 Homoclinic Tangencies

Organizers : Ale Jan Homburg, Floris Takens

Key note lecture

Recent developments in chaotic dynamics

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I will report on some recent results in the theory of dynamical systems with complex behaviour, especially on the properties of attractors, robustness and (non)hyperbolicity, statistical description of the dynamics, and stability.

Invited lectures

Periodic and strange attractors near saddle-focus homoclinic orbits

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We discuss dynamics near Shil'nikov saddle-focus homoclinic orbits in three dimensional vector fields. The existence of periodic and strange attractors is investigated in families for which each member has a homoclinic orbit (so not in unfoldings). We consider how often, in the sense of measure, periodic and strange attractors occur. We also discuss the fate of typical orbits, and establish that despite the possible existence of attractors, a large proportion of points lies outside the basin of an attractor. This is joint work with Sergey Gonchenko and Oleg Sten'kin.

The Lorenz attractor exists

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The following non-linear system of differential equations, also known as the *Lorenz equations*,

$$\begin{aligned} \dot{x}_1 &= -\sigma x_1 + \sigma x_2 \\ \dot{x}_2 &= \varrho x_1 - x_2 - x_1 x_3 \\ \dot{x}_3 &= -\beta x_3 + x_1 x_2, \end{aligned} \tag{1}$$

was introduced 1963 by Edward Lorenz, see [1]. Numerical simulations for an open neighbourhood of the classical parameter values $\sigma = 10, \beta = 8/3$ and $\varrho = 28$ suggest that almost all points in phase space tend to a strange attractor $\mathcal{A}$ - *the Lorenz attractor*. In [2], we prove that the Lorenz equations indeed support a strange attractor, as conjectured in [1]. We also prove that the attractor is robust, i.e., it persists under small perturbations of the coefficients in the underlying differential equations:

Theorem: *For the classical parameter values, the Lorenz equations support a robust strange attractor $\mathcal{A}$. Furthermore, the flow admits a unique SRB measure μ_X with* $\mathrm{supp}(\mu_X) = \mathcal{A}$.

The proof can be broken down into two main sections: one global part, which involves rigorous numerical computations, and one local part, which is based on normal form theory. The novelty of the method of proof lies in that, rather than producing a traditional mathematical proof, we construct an algorithm which, if successfully executed, proves the existence of the strange attractor.

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Arbitrarily complicated dynamics near homoclinic tangencies

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We prove that any multidimensional dynamical systems with non-hyperbolic homoclinic and heteroclinic cycles exhibits all dynamical phenomena possible in the corresponding dimension, in the following sense. Given a C^r-diffeomorphism f in R^n with a heteroclinic (homoclinic) cycle C composed of a finite number of periodic and connecting orbits there exists an algorithm to evaluate an effective dimension d of the problem: the maximal possible number of zero Lyapunov exponents for periodic orbits which can be born in a small neighbourhood of the cycle C at small C^r-smooth perturbations of f. For the case $d < n$ the following result is established:

Theorem. *For any d-dimensional diffeomorphism g and for any ε there exists a diffeomorphism f^* arbitrarily close to f in C^r-topology such that f^* has an absorbing domain D (i.e., $(f^*)^N D \subset D$ for some positive N) in a small neighbourhood of the cycle C, and in D there is an attractive d-dimensional smooth invariant manifold M such that $(f^*)^N|_M$ is C^r-conjugate to g.*

Roughly speaking, small perturbation of any system with a heteroclinic cycle of effective dimension d can produce arbitrary d-dimensional dynamics.

Contributed talks

Control of averaging chaos in quasiperiodically excited system

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Suppression of chaos in periodically forced nonlinear oscillators has received much effort from analytical, numerical and experimental point of view [1,2,3]. In this work suppression of chaos in the averaged system of a nonlinear one-degree- of-freedom oscillator subjected to external and parametric excitations having incommensurate frequencies is reported [4]. An adapted Bogolioubov-Mitropolski method [5] is first developed to reduce the original quasiperiodically driven system to a periodically driven one by averaging in the rapidly varying oscillations [6]. The chaotic threshold is then derived on the slowly varying oscillations using the Melnikov method [7]. The possibility to realize a suitable system control of chaos by introducing a third harmonic parametric component into a nonlinear term of the original quasiperiodically forced system is investigated by acting on the induced effective averaged equations.

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Homoclinic tangencies and Ω-moduli

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As is well known, multidimensional dynamical systems with structurally unstable Poincare homoclinic orbits and heteroclinic cycles possess continuous invariants (moduli) of the topological or Ω-equivalence (conjugacy) (the last is the topological equivalence (conjugacy) on the set of nonwandering orbits). For some classes of such systems the principal Ω-moduli were found and the existence of infinitely many moduli was established (see, for example, [1]–[4]). The importance of calculation of principal Ω-moduli is explained by the fact that, by definition, these invariants represent such parameters whose arbitrary changes lead to bifurcations of nonwandering orbits (periodic, homoclinic etc.). Thus, Ω-moduli can be considered as substantive control parameters along with the splitting parameters which are natural for such type bifurcation problems.

In the present report we state some results related to the following problems:
1) Determination of principal Ω-moduli in the cases of multidimensional diffeomorphisms with structurally unstable homoclinic orbits or heteroclinic cycles. A special attention is given for the cases where weakest eigenvalues of saddles are complex conjugate. Also we consider questions on existence of moduli of local Ω-conjugacy in the case of symplectic maps.
2) The study of bifurcations for families whose control parameters are a) principal Ω-moduli ; b) both principal Ω-moduli and splitting parameters.

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Asymptotics of Homoclinic bifurcation in a Three-Dimensional systemt

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Using perturbation methods, we predict critical parameter values corresponding to homoclinic bifurcation in three-dimensional systems. The multiple scales perturbation method [1] is first performed to construct a higher-order approximation of the periodic solution [2]. A criterion based on the collision between the periodic orbit and the equilibrium involved in the bifurcation is then applied. This criterion developed initially to predict homoclinic bifurcations in planar autonomous system [3,4,5] is adapted here to derive a critical value of the homoclinic bifurcation in two specific three-dimensional systems. Period- doubling bifurcations are also investigated using the method of harmonic balance and the stability analysis of the bifurcating periodic solution. A numerical study [6,7] showed a good agreement of the critical parameter value with this analytical prediction.

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Dynamics near non-transversal homoclinic points

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We consider discrete dynamical systems $x(n+1) = f(x(n), \lambda)$ having for $\lambda = 0$ a homoclinic orbit $\Gamma := \{f^n(q,0), n \in \mathbb{Z}\}$ asymptotic to a hyperbolic fixed point p. $f : \mathbb{R}^k \times \mathbb{R}^m \to \mathbb{R}^k$ smooth. We are interested in the dynamics in a neighborhood of Γ. To investigate this dynamics we will use Lin's method. In the theory of smooth dynamical systems (vector fields) this method is proved to be a powerful tool for investigating the dynamics near homoclinic orbits or even heteroclinic cycles. Here we carry forward the ideas of Lin's method to discrete systems. We apply the results to study the dynamics near non-transversal homoclinic points:

If the stable and unstable manifolds of p have a quadratic tangency in q we give a "Poincaré-map" involving two different transversal homoclinic points which is conjugated to shift dynamics. We consider the dynamics of this map in dependence on the parameter λ.

By means of the canonical Poincaré-map we study the dynamics near homoclinic points in periodically forced systems. In particular we study homoclinic points and subharmonic solutions.

Resonant homoclinic flip bifurcations: a numerical investigation

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Three-parameter unfoldings of resonant orbit flip and inclination flip homoclinic orbits have recently been studied in [2]. The key idea is to look at a sphere in parameter space around the central singularity and glue respective codimension-two unfoldings of homoclinic flip bifurcations together. Several cases are possible, and near some resonant flip bifurcations one can find homoclinic-doubling cascades as first found in [1].

We explain some of these unfoldings and show results of a careful numerical study of the model proposed by Sandstede in [3]. Two particular unfoldings of a resonant orbit flip and a resonant inclination flip do indeed occur as proposed in [2] when the sphere around the central singularity is taken small enough. This also confirms the conjecture that the parameter space has cone structure close to the central codimension-three point. When the sphere is taken larger extra interesting codimension-three bifurcations occur.

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On the structure and bifurcations in a generic unfolding of a 2 d.o.f Hamiltonian system with the contour with two saddle-foci

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Let (M, Ω) be a C^∞ four-dimensional symplectic manifold and $X_{H_\mu},\ \mu \in \mathbb{R}$, be a generic smooth one-parameter unfolding of a smooth Hamiltonian vector field with Hamiltonian H_0 such that X_{H_0} has two singular points p_1, p_2, both of saddle-focus type that belong to the same level set of Hamiltonian and their stable and unstable manifolds intersect each other transversely along two heteroclinic orbits forming a contour Γ. Without loss of generality one may suppose the equilibrium p_1 does not change with μ and p_2 does depend on μ, with $f(\mu) = H(p_2) - H(p_1)$.

Theorem. There are positive constants $\mu_0,\ c_0$ small enough and a neighborhood U of Γ such that for $X_{H_\mu},\ |\mu| \le \mu_0$, in $U \cap \{H_\mu = c\},\ |c| \le c_0$: 1. There exist invariant hyperbolic subsets $\mathcal{H}_{\mu,c}$ such that the Poincare map $P_{\mu,c}$ generated by

$\mathcal{H}_{\mu,c}$ is conjugated to a symbolic system with countable alphabets for $\mu = 0,\ c = 0$, $\mu \ne 0,\ c = 0$ or $\mu \ne 0,\ c = f(\mu)$. If $\mu \ne 0,\ c \ne f(\mu)$ then the alphabet is finite and the number of states grows with the logarithmic asymptotics. 2. There are two sequences

$\{\mu_k^{(1)}\}$, $\{\mu_k^{(2)}\}$, $\mu_k^{(i)} \in (-\mu_0,\ \mu_0)$, such that X_{H_μ}, $\mu = \mu_k^{(i)}$, possesses a nontransverse homoclinic orbit to p_i with quadratic tangency. After every passage of μ through $\mu_k^{(i)}$ the number of 1-circuit homoclinic orbits to p_i increases by 2. 3. For $\mu = c = 0$ the set $\mathcal{H}_{\mu,c}$ exhausts all orbits lying entirely in U, in particular, all homoclinic and heteroclinic to p_i, $i = 1, 2$, orbits.

Corollary 1. The vector field X_{H_0} in the level $H_0 = 0$ near Γ has:

i) a countable set of transverse heteroclinic contours of any circuitness;

ii) each saddle-focus p_i has a countable set of transverse homoclinic orbits of any circuitness;

iii) for each integer $N > 0$ there is a countable set of saddle periodic orbits of circuitness N, every such orbit γ possesses a countable set of heteroclinic orbits with p_i, $i = 1, 2$, $\gamma \to p_i$, and $p_i \to \gamma$, where the arrows point out the direction of increasing time.

Corollary 2. For $\mu \neq 0$, $c = 0$ the same assertions as in the items ii), iii) of Corollary 1 hold for p_1 and periodic orbits in the level $H_\mu = 0$. For $\mu \neq 0$, $c = f(\mu)$ the same is valid for the saddle-focus $p_2(\mu)$.

Remark. The contours described exist in the Hamiltonian system derived from the stationary generalised Swift-Hohenberg equation when its two parameters belong to some parabola.

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On singularities robustly accumulated by periodic orbits

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Let $\mathcal{G}^1$ be the set of three-dimensional vector fields X such that all the singularities and periodic orbits of every vector field Y C^1-close to X are hyperbolic. A singularity σ of X is *robustly accumulated by periodic orbits* if for every Y C^1 close to X, the continuation σ_Y of σ belongs to the closure of the periodic orbits of Y. An example of such a kind of singularity is that of the geometric Lorenz attractor L (cf. [1]). Observe that L is a partially hyperbolic attractor with volume expanding central direction.

In this talk we outline the proof of the following result : For *generic* $X \in \mathcal{G}^1$, if σ is a singularity of X robustly accumulated by periodic orbits, then σ belongs to a partially hyperbolic attractor with volume expanding central direction for either X or $-X$. An example of a vector field in $\mathcal{G}^1$ which has neither attractors nor repellers and exhibits a singularity robustly accumulated by periodic orbits is given.

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Impossibility of complete bifurcation description for some classes of systems with simple dynamics

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The main problem of the bifurcation theory is to describe changes of phase portraits of dynamical systems under continuous change of parameters. This description implies construction of the bifurcation diagram, i.e. splitting the space of parameters onto classes of the equivalent systems. It is well known, that many bifurcations of dynamical systems admit the complete bifurcation description by so-called versal families, which, in some sense, contain the complete information on the structure of the bifurcation set for any family of general position. it is known also that majority of dynamical systems with chaotic behaviour of trajectories do not admit the complete bifurcation analysis. It is explained, first of all, by the existence of small perturbations which lead to appearance of infinitely-degenerate orbits. In contrast to the chaotic systems, it seems natural, that systems with simple dynamics must admit the complete bifurcation analysis. But it is not so.

In the present report, an example of the codimension two system with very simple dynamics is considered. Small perturbations of this system can lead to appearance of no more than one periodic orbit. Nevertheless, the complete bifurcation analysis of such system is impossible in framework of any finite parametric family of dynamical systems.

More exactly, bifurcations of a heteroclinic contour composed by a homoclinic orbit to a saddle-focus and a heteroclinic orbit to this equilibrium point are considered. The case of simple dynamics is selected (no more than one periodic orbit is born at bifurcations in a small neighborhood of the contour). In spite of simplicity of dynamic behavior, the structure of the bifurcation set corresponding to multi-circuit heteroclinic orbits is rather complicated. It is demonstrated that there is no versal families of dynamical systems which describe completely the structure of the bifurcation set. Moreover, in any neighborhood of any family with such contour there exists a continuum set of the families which are not equivalent to each other.

A Method for Computing the Homoclinic Orbits of the Lorenz System at $\sigma = 10, \mathbf{b} = 8/3$ and $\rho = 13.926\ldots$

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We present an iterative technique to analytically approximate the homoclinic loops of the Lorenz system for $\sigma = 10, b = 8/3$ and $\rho = \rho_H = 13.926\ldots$. First, the local structure of the homoclinic solution for $t \longrightarrow 0\pm$ and $t \longrightarrow \pm\infty$ is analyzed. Then, global approximants are used to match the local expansions. The matching procedure resembles the one used in Padé approximations. The accuracy of the approximation is improved iteratively, with each iteration providing estimates for the initial conditions of the homoclinic orbit, the valus of ρ_H, and three undetermined constants in the local expansions. Within three iterations the error in ρ_H falls to the order of 0.1%. Comparisons with numerical integrations are made, and a discussion on ways to extend the technique to other types of homoclinic or heteroclinic orbits, and to improve its accuracy, is given. A detailed presentation of the aforementioned analysis can be found in [1].

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1.3 Singular Perturbations

Organizers : Peter Szmolyan, David Terman

Key note lecture

Geometric Singular Perturbation Theory

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The geometric approach to singular perturbation problems [1] has been substantially refined and extended during the last decade. For problems which exhibit layer behaviour this approach has been very successful, see the survey [2] for the basic theory and references to numerous applications. By now, singularly perturbed ordinary differential equations can be seen as dynamical systems of considerable complexity but still amenable to analysis.

However, at points where normal hyperbolicity fails, e.g. fold points or points of self-intersection of the critical manifold, the well developed geometric theory does not apply. These situations are abundant in applications, we just mention relaxation oscillations [3] and turning point problems [4]. A well known related phenomenon are canard solutions which were first analysed by methods from nonstandard analysis [5] . Only recently, a pioneering geometric analysis of canard cycles in van der Pol's equation based on blow-up methods was given [6].

In this talk we survey results [7,8,9,10] aiming at a comprehensive geometric treatment of singular perturbation problems with nonhyperbolic points. We show how blow-up techniques lead to problems which can be analyzed in detail by more conventional methods from the theory of dynamical systems. Several examples, including relaxation oscillations and canard solutions, are discussed. This approach provides also insight into the scaling- and matching-techniques used in classical matched asymptotic expansions.

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Invited lectures

Geometric singular perturbation theory beyond normal hyperbolicity

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Geometric singular perturbation theory has traditionally dealt only with perturbation problems near normally hyperbolic manifolds of singularities. In the talk will be shown how blow up techniques can permit enlarging the applicability to non-normally hyperbolic points. The method will be presented on well chosen examples in the plane and in 3-space, emphasizing problems concerning bifurcations of limit cycles.

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Geometric singular perturbation theory in infinite dimensions

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Geometric singular perturbation theory is a powerful tool for the study of dynamical systems with two different time scales. However, its basic techniques, originally established by N. Fenichel, are strictly finite-dimensional in nature. This talk surveys recent infinite-dimensional developments in the framework of two examples. First, Shilnikov-type manifolds homoclinic to finite-dimensional slow manifolds are described for coupled nonlinear Schrődinger equations. Second, the existence of an infinite-dimensional slow manifold comtaining the global attractor of the Maxwell-Bloch equations is discussed.

Asymptotic Method of Differential Inequalities

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Some recent achievements of singular perturbations theory which are based on asymptotic method of differential inequalities (AMDI) are presented.

The main idea of AMDI is to construct upper and lower solutions by using formal asymptotics. This approach was successfully applied to different classes of problems for nonlinear singularly perturbed equations:

1) Elliptic problems, including systems with "fast" and slow equations.

2) Parabolic problems with solutions periodic in time.

3) Integro-differential equations (ODE and PDE cases).

4) Investigations of stability in Ljapunov sense of steady-state and periodic solutions of the corresponding parabolic equations.

For these classes of problems solutions with internal layers and the case of exchange of stabilities were considered. Theorems of existence of solutions, estimates of asymptotics were stated. More detailed consideration of some of the presented problems can be found in papers [1-3].

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Captures into resonance and scattering on resonance: an example from charged particles dynamics

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Dynamics of a relativistic charged particle in a uniform magnetic field and an electrostatic plane wave is studied. For small amplitude high frequency wave the system is reduced to a two degrees of freedom Hamiltonian system with slow and fast variables. In this system, the phenomena of capture into resonance and scattering on the resonance are described. These phenomena determine the long-term dynamics of the particle.

Gevrey Asymptotics in Singular Perturbations of ODE

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For analytical singularly perturbed ordinary differential equations, I present two very general theorems on the existence of solutions having Gevrey asymptotic expansions and an idea of their proof (work with M. Canalis–Durand, J.-P. Ramis and Y. Sibuya).

Three applications are given:

- the proof of Wasow's conjecture on the nature of turning points (thesis of C. Stenger),
- a theorem on resonance in the sense of Ackerberg - O'Malley (work with A. Fruchard),
- a generalisation of a theorem of Y. Sibuya on the convergence of formal solutions in the theory of singular perturbations.

The theorems were chosen to show the power of *analytical* methods in the theory of singular perturbations.

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Propagating Activity Patterns in Neuronal Networks

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Synaptically coupled neuronal networks can exhibit a wide variety of propagating wave activity. These waves depend on many network properties including whether the coupling is excitatory or inhibitory and the architecture of neuronal connections. In this talk, I will discuss how geometric singular perturbation methods have been used to classify which waves exist for a given network and how to compute their wave speed.

Contributed talks

Bifurcations, scaling laws and hysteresis in singularly perturbed systems

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Dynamical systems depending on a slowly varying parameter can be described by a singularly perturbed differential equation of the form $\varepsilon\dot{x} = f(x,t)$. We are interested in the qualitative behaviour of solutions near a bifurcation point of the associated one-parameter family of dynamical systems $\dot{x} = f(x,\lambda)$.

Bifurcations with a single zero eigenvalue lead to the appearance of scaling laws: certain solutions of the singularly perturbed equation track an equilibrium branch of $f(x,\lambda)$ at a distance of order $\varepsilon^{\alpha}|t|^{\beta}$. When $f(x,t)$ is periodic, this can produce hysteresis cycles whose area behaves as ε^{μ}, where μ is usually a rational number. We present a method based on Newton's polygon to compute the exponents α, β and μ [1,2].

The passage through a Hopf bifurcation leads to the well-known bifurcation delay [3,4]. We construct a scalar feedback control which suppresses this delay by transforming the Hopf bifurcation into a bifurcation with double zero eigenvalue, described by a cubic Liénard equation [5]. We present a result on exchange of stabilities for the associated slowly time-dependent equation [6].

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Exponential Sensitivity in some singularly perturbed boundary value problems

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We consider a singularly perturbed boundary value problem with Dirichlet conditions and study the sensitivity of the internal layers solutions with respect to small changes in the boundary data. Our approach exploits the existence of smooth invariant manifolds and their asymptotic expansions in the small parameter of perturbation. Using this approach we show that the phenomenon is extremely sensitive since the shock layers are only obtained by exponentially small perturbations of the boundary data.
Examples considered include certain equations associated with metastable internal layer motion.

Integral Manifolds in the Theory of Singularly Perturbed Differential-Difference Equations

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We consider the following system with time lag

$$\begin{aligned} \frac{dx}{dt} &= f(t,x,y,\varepsilon), \\ \varepsilon\frac{dy}{dt} &= C(t)y + D(t)y_\Delta + g(t,x,x_\Delta,y,y_\Delta,\varepsilon), \end{aligned} \tag{1}$$

where $t \in \mathbb{R}$, $x, x_\Delta \in \mathbb{R}^n$, $y, y_\Delta \in \Omega_\rho = \{y \in \mathbb{R}^m, |y| < \rho\}$, $\varepsilon > 0$ $(x_\Delta = x(t-\varepsilon\Delta), y_\Delta = y(t-\varepsilon\Delta))$.

In this paper we investigate conditions of existence for an integral manifold of system (1) in form

$$y = h(t,x,\varepsilon).$$

The flow on this manifold is governed by the m-dimentional system

$$\frac{dx}{dt} = f(t,x,h(t,x,\varepsilon),\varepsilon). \tag{2}$$

Moreover, the stability property problem for system (1) is equivalent to the stability problem for reduced system (2) [1].

Method of integral manifolds is applied to study linear singularly perturbed system [2]

$$\begin{aligned} \frac{dx}{dt} &= A(t)x + B(t)y, \\ \varepsilon\frac{dy}{dt} &= C(t)y + D(t)y_\Delta + F(t)x + G(t)x_\Delta. \end{aligned} \tag{3}$$

The use of this method permits us to solve a problem of decomposition of singularly perturbed differential-difference system (3) into two independent equations.

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The period function of non homogeneous equations

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We study the period function $T(c)$ of a center of non homogeneous differential system:

$$u'' + g(t, u, u') = 0, \tag{1}$$

where the function g satisfies certain conditions, ensuring the existence of a center at the origin O.

We are interested in the existence of solutions of equation (1), and more precisely in the case where $g(t, u, u') = f(u) + \epsilon h(t, u, u')$. So that the homogeneous equation $u'' + f(u)$ has a center and a monotonic period function depending on the energy. Some sufficient conditions for the monotonicity of $T(c)$ or for the isochronocity of a center in the homogeneous case may be extended in the perturbed case.

We also examine the differential equation of Lienard type $u'' + a(t)u' + f(u) = 0$, and the monotonicity of its period function.

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Invariant manifolds approach to optimal control of nonlinear singularly perturbed systems

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Consider the following nonlinear affine system:

$$\dot{x}_1 = f_1(x_1, x_2) + B_1(x_1, x_2)u, \quad \epsilon\dot{x}_2 = f_2(x_1, x_2) + B_2(x_1, x_2)u, \tag{1}$$

with the functional

$$J = \int_0^\infty [k'(x_1, x_2)k(x_1, x_2) + u'R(x_1, x_2)u]dt, \tag{2}$$

where $x_1(t) \in R^{n_1}$ and $x_2(t) \in R^{n_2}$ are the state vectors, $x = \text{col}\{x_1, x_2\}$, $u(t) \in R^m$ is the control input, and $z \in R^s$ is the output to be controlled, $\epsilon > 0$ is a small parameter.

The functions f_i, B_i, R and k are differentiable with respect to x a sufficient number of times. We assume also that $f_i(0,0) = 0, k(0,0) = 0$ and $R = R' > 0$.

The system (1)-(2) has a non-standard singularly perturbed form in the sense that it is nonlinear in both, the slow variable x_1 and the fast variable x_2. In the standard form the system is linear in x_2.

We are looking for a nonlinear state-feedback

$$u = \beta(x), \quad \beta(0) = 0, \tag{3}$$

that minimizes the cost (2), where $x(0) = x_0$. For each $\epsilon > 0$ the control law (3) is locally optimal on $\Omega \subset R^{n_1} \times R^{n_2}$ if there exists Ω_1, $0 \in \Omega \subset \Omega_1$, such that the closed-loop trajectories for initial data in Ω remain in Ω_1 and for any initial condition $x_0 \in \Omega$ and any control $u(t)$ for which

$$(i)\ x(t) \in \Omega_1,\ t \geq 0; \quad (ii)\ J(x_0, u) < \infty; \quad (iii) \lim_{t\to\infty} x(t) = 0$$

we have $J(x_0, u_0) \leq J(x_0, u)$.

It is known that the optimal controller for such problem can be designed by solving a Hamilton-Jacobi partial differential equation. This equation is solvable iff there exists a special invariant manifold of the corresponding Hamiltonian system. We obtain exact slow-fast decomposition of the Hamiltonian system and of the special invariant manifold into the slow and the fast ones. On the basis of this decomposition we construct high-order asymptotic approximations of the optimal state-feedback, optimal trajectory and optimal open-loop control.

Multiplicative solution asymptotics for singularly perturbed parabolic boundary value problems

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In this talk we consider the boundary value problem

$$\frac{\partial u}{\partial t} = L_h u, \quad t > 0, \quad x \in D; \tag{1}$$

$$u\Big|_{t=0} = 0, \quad u\Big|_{x\in\partial D} = \Psi(x), \tag{2}$$

where $u = u(x, t, h)$,

$$L_h = h\langle \mathbf{A}(x)\nabla, \nabla\rangle + \langle \mathbf{b}(x), \nabla\rangle + c(x)$$

is an operator uniformly parabolic on the set D_0, $\mathbf{A}(x)$ is a symmetric $(n \times n)$ matrix, $\mathbf{b}(x) = (b_1(x), b_2(x), \ldots, b_n(x))$ is a vector-valued function, the components of the matrix $\mathbf{A}(x)$ and of the vector $\mathbf{b}(x)$ are infinitely differentiable on D_0; the function $c(x) \leq 0$ for $x \in D_0$, and $c(x) \in C^\infty(D_0)$; D_0 is a bounded domain such that $(D \cup \partial D) \subset \mathbb{R}^n$; the function $\Psi(x) \geq \delta > 0$ for $x \in \partial D$, $\Psi(x) \in C^\infty(\partial D)$. The boundary ∂D of the domain D is a class C^∞; $h \in (0,1]$ is a small parameter, $\langle \cdot, \cdot \rangle$ is the inner product in $\mathbb{R}^n_x$, $\nabla = (\partial/\partial x_1, \partial/\partial x_2, \ldots, \partial/\partial x_n)$ is the gradient operator.

We construct the global exponential asymptotics of the solution of problem (1),(2) as $h \to +0$ for a finite time interval. We investigate following three cases:

$$(i)\ \langle \mathbf{b}(x), \mathbf{n}(x) \rangle > 0; \quad (ii)\ \langle \mathbf{b}(x), \mathbf{n}(x) \rangle < 0; \quad (iii)\ \langle \mathbf{b}(x), \mathbf{n}(x) \rangle = 0,$$

where $x \in \partial D$ and $\mathbf{n}(x)$ is the unit outward normal vector to ∂D at the point x.

We also construct the global asymptotics of the solution of problem (1), (2) in the case in which the equation coeffients of L_h depend on x and t.

Part of these results are joint work with V.P. Maslov and S.I. Chernykh.

Limit Cycles of Polynomial Dynamical Systems

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Two-dimensional polynomial dynamical systems are considered. The main problem of the qualitative theory of such systems is Hilbert's Sixteenth Problem on the maximum number and relative position of limit cycles. There are three local bifurcations of limit cycles: 1) Andronov – Hopf bifurcation (from a singular point of the center or focus type); 2) separatrix cycle bifurcation (from a homoclinic or heteroclinic orbit); 3) multiple limit cycle bifurcation. The analysis of each of these bifurcations individually does not yield a complete solution of the Problem even in the simplest (quadratic) case of nonlinear polynomial systems. Using Erugin's ideas on the qualitative investigation on the whole, Duff's field-rotation parameters and Perko's termination principle, we connect all local bifurcations of limit cycles and develop a global bifurcation theory of polynomial dynamical systems.

In particular, we give a sketch of proof of the theorem stating that for quadratic systems four is the maximum number of limit cycles and (3:1) is only possible their distribution. The proof is carried out by contradiction. On the first step we show the nonexistence of four limit cycles surrounding a singular point. We consider a canonical system containing three field-rotation parameters and suppose that it has four limit cycles around the origin; then we get into some three-dimensional domain of the field-rotation parameters being restricted by some conditions on the rest two parameters corresponding to the definite case of singular points in the phase plane. This three-parameter domain of four limit cycles is bounded by three fold bifurcation surfaces forming a swallow-tail bifurcation surface of multiplicity-four limit cycles. It can be shown that the corresponding maximal one-parameter family of multiplicity-four limit cycles cannot be cyclic and terminates either at the origin or on some separatrix cycle surrounding the origin. Since we know absolutely precisely at least the cyclicity of the singular point (Bautin's result) which is equal to three, we have got a contradiction with the termination principle stating that the multiplicity of limit cycles cannot be higher than the multiplicity (cyclicity) of the singular point in which they terminate. Since we know the concrete properties of all three field-rotation parameters in the canonical system and, besides, we are able to control simultaneously bifurcations of limit cycles around different singular points, the proof of the theorem can be completed.

In a similar way cubic and more general polynomial systems can be considered. Thus, generalizing the obtained results, we develop a global bifurcation theory of planar polynomial systems. We discuss also how to apply these results for higher-dimensional dynamical systems and how, in particular, to construct a three-dimensional system with a strange attractor on the base of a planar quadratic system with two unstable foci and an invariant straight line.

On WKB asymptotic expansions of high frequency vibrations in stiff problems

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Stiff problems are differential equation problems with very different values of the coefficients in different regions of the domain. Such problems model the vibrations of a system consisting of two (or more) materials, one of them very stiff respect to the other. Stiff problems originated in J.L. Lions [1]. We study the asymptotic behavior of the eigenvalues and eigenfunctions of the corresponding spectral problem.

Let Ω be a bounded domain of $\mathbb{R}^n$, $n = 1, 2, \ldots$, divided in two parts Ω_0 and Ω_1 by the interface Γ: $\Omega = \Omega_0 \cup \Omega_1 \cup \Gamma$. Consider the spectral boundary value problem

$$L_0(x, \partial_x)u^\varepsilon - \lambda^\varepsilon u^\varepsilon = 0 \quad \text{in } \Omega_0 \quad \text{and} \quad \varepsilon L_1(x, \partial_x)u^\varepsilon - \lambda^\varepsilon u^\varepsilon = 0 \quad \text{in } \Omega_1$$

with the Dirichlet boundary conditions on $\partial\Omega$ and the standard conjugate conditions on Γ. Here L_0 and L_1 are elliptic operators. The asymptotic behavior as $\varepsilon \to 0$ of eigenvalues λ_k^ε and eigenfunctions u_k^ε has been widely studied (see [2] and the references given there). The study of so called *low frequencies* $\lambda_k^\varepsilon = O(\varepsilon)$ leads us to a Dirichlet eigenvalue problem posed in the less stiff structure Ω_1 and does not provide a good insight on the vibration problems over all Ω.

From a physical viewpoint there is the different kind of proper vibrations (so called *high frequency vibrations*) which correspond to sequences of eigenvalues $\lambda_{k(\varepsilon)}^\varepsilon$ with $k(\varepsilon) \to \infty$. Such vibrations are generated by the stiffer structure Ω_0 and have strongly oscillatory character in Ω_1 (see [3]). Constructing of asymptotic expansions of the high frequency vibrations has been an open problem. Use a WKB-technique we find for some operators L_i and domains Ω_i the full asymptotic expansions of such vibrations in the form

$$u^\varepsilon(x) \sim \sum \varepsilon^k v_k(x) \quad \text{if } x \in \Omega_0, \quad u^\varepsilon(x) \sim \sum \varepsilon^k f_k(x, \varepsilon^{-1}S(x)) \quad \text{if } x \in \Omega_1$$

where S is a solution of the corresponding Hamilton-Jacobi equation. The asymptotics has a discrete character with respect to a small parameter ε. The sequences $\varepsilon_s \to 0$ on which expansions are constructed and justified are given by formula.

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Radiation Reaction and Center Manifolds

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We consider a charged particle subject to self-generated electric and magnetic fields and under the influence of external fields, the classical Abraham-Lorentz model. An important issue for such models is to derive an "effective equation" for the particle position $q(t) \in \mathbb{R}^3$, i.e., an ODE for q alone, with the contributions from Maxwell's equations for the fields incorporated. By formal calculations it can be seen that this equation is of type

$$(m_0 + \varepsilon^{-1} m_e)\ddot{q} = F(q, \dot{q}) + a q^{(3)}, \tag{1}$$

with $a > 0$, m_0 and m_e the rest mass and the effective mass of the particle, respectively; F denotes the external forces. This is the famous Lorentz-Dirac equation which (due to the term with the third derivative) has undesirable "run-away"-solutions, i.e., solutions that go off to infinity exponentially fast. There have been a lot of attempts to single out those initial data that lead to physically relevant solutions of (1) and to interpret them as approximations of the full problem, but no satisfactory answer has been know.

In this talk we outline a complete solution to this problem (and a rigorous derivation of (1)). It will be shown that the physically relevant solutions are exactly those which build up a normally hyperbolic manifold for (1) as a singular perturbation problem. In addition, those solutions are good approximations of the solutions to the full Abraham-Lorentz equations. The true "effective equation" then is the corresponding equation on the center-like manifold, and this equation has only physically reasonable solutions. (Joint work with H. Spohn.)

A perturbation method based on integrating vectors

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In this paper a perturbation method based on integrating vectors and multiple scales will be presented for initial value problems for regularly or singularly perturbed ordinary differential equations. Asymptotic approximations of first integrals will be constructed which are valid on long time-scales. In some cases approximations of first integrals can be obtained that are valid for all times. Several examples are treated to show how this perturbation method can be applied.

On the asymptotic behaviour of solutions of singular initial value problems

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The following singular initial value problem is under consideration

$$\alpha(t)x'(t) = F(t, x(t), x'(t)), \tag{1}$$

$$x(0) = 0, \tag{2}$$

where $\alpha : (0,\tau) \longrightarrow R^{n\times n}$ - a matrix with continuous elements,

$$\alpha(t) = \mathrm{diag}(\alpha_1(t), \ldots, \alpha_n(t)),$$

$\alpha_i(t) \to 0, \quad t \to +0, \quad i \in \{1,\ldots,n\},$ F is a continuous function in some domain. For any $\rho \in (0,\tau)$ ρ – solution of the problem (1), (2) is said to be a continuously differentiable function $x : (0,\rho) \longrightarrow R^n$ which is satisfield (1) when $t \in (0,\rho)$ and also $x(t) \to 0$ when $t \to +0$. Sufficient conditions are given wherein the problem (1), (2) has a nonempty set of ρ - solutions (where $\rho > 0$ is small sufficiently) with needed asymptotic properties when $t \to +0$. In particular, special cases $\alpha_i(t) = \beta(t)$, $i \in \{1,\ldots,n\}$ and $\alpha_i(t) = \beta_1(t)$, $i \in \{1,\ldots,n_1\}$, $\alpha_j(t) = \beta_2(t)$, $j \in \{n_1+1,\ldots,n\}$ are under consideration. New efficient algorithms have been designed. All theorem's conditions are effective. The investigation methods can be used for solving of a wide range of problems of local analysis.

1.4 Stochastic Systems

Organizers : Ludwig Arnold, Peter Kloeden

Key note lecture

White Noise Eliminates Instability

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For deterministic systems with nontrivial limit behaviour the concept of a global attractor has turned out to be useful. A global attractor is a compact invariant subset of the state space, which attracts every bounded set.
When taking additional random influences into account, the concept of a global attractor has to be extended. One possible approach is to allow the attractor to be a random

set. A compact random set is said to be a *random attractor* if it is invariant in a suitable sense, and if it attracts every bounded deterministic set. It turns out that random attractors exist for several classes of systems, among them random perturbations of systems from mathematical physics.
When considering random influences of small intensity, the question of the relation between the attractor of the unperturbed system and the family of random attractors of the randomly perturbed systems arises. It turns out that, with the intensity of the perturbation tending to zero, the random attractor stays in arbitrarily small neighbourhoods of the unperturbed attractor almost surely.
However, in general the random attractor does *not* converge to the unperturbed attractor. Examples show that random perturbations may cause the random attractor to become *smaller* than the unperturbed attractor, where 'smaller' not only refers to set inclusion, but also to (Hausdorff) dimension.
A possible explanation of this phenomenon is the fact that the perturbed system becomes transient in a vicinity of unstable regions of the attractor of the unperturbed system, even if the intensity of the random perturbation is very small.

Invited lectures

Bifurcation theory for stochastic differential equations

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In many physical systems which can be modelled by Itô stochastic differential equations, the equilibrium and stability properties of the system change as certain parameters in the system vary. Stationary probability measures may appear or disappear; (almost surely) stable fixed points may become (almost surely) unstable; pairs of trajectories may diverge (almost surely) instead of converging. Very often these phenomena are associated with the change of sign of a relevant Lyapunov exponent. In this talk we shall give some examples and results concerning such systems.

Random homoclinic dynamics

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Transversal random homoclinic points play an important role for the description of smooth random dynamical systems exhibiting chaotic behaviour. In discrete time their existence gives rise to an invariant random homoclinic set on which the dynamics can be modeled by a random subshift of finite type. Following [1] we present a corresponding random version of the Smale Birkhoff Theorem which pays tribute to this phenomenon.

In continuous time a random Melnikov method can be used to have a recourse to the result in discrete time. We describe a method from [2] where criteria are provided in terms of simple zero-crossings of a Melnikov process for a randomly perturbed differential equation on $\mathbb{R}^p$ of the form

$$\dot{x} = f(x) + \epsilon g(\theta(t)\omega, x, \epsilon)$$

where f is a bounded C^3 function such that this equation possesses a hyperbolic fixed point x_0 for $\epsilon = 0$ and a homoclinic solution $\zeta : \mathbb{R} \to \mathbb{R}^p$ corresponding to x_0, i.e. $\lim_{t\to\pm\infty} \zeta(t) = x_0$, $(\theta(t))_{t\in\mathbb{R}}$ is an ergodic metric dynamical system on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with Ω being a topological space and $t \mapsto \theta(t)\omega$ being continuous for all $\omega \in \Omega$, $g : \Omega \times \mathbb{R}^p \times (-\epsilon_0, \epsilon_0) \to \mathbb{R}^p$ for some $\epsilon_0 \in \mathbb{R}_+$ is a measurable, uniformly bounded function which is C^3 for each fixed $\omega \in \Omega$ and continuous for each fixed (x, ϵ). In case of $p = 2$ and the unperturbed system being Hamiltonian, the Melnikov process can be nicely presented as

$$t \mapsto M(\theta(t)\omega) = \int_{-\infty}^{\infty} f(\zeta(s-t)) \wedge g(\theta(s)\omega, \zeta(s-t), 0)ds$$

and hence consists just of given quantities such that it can be, in principle, computed. Steinkamp's Melnikov criterion requires that the number of simple zero-crossings of the Melnikov process has positive expectation. If this is satisfied, then a Poincaré section can be constructed that leads to a random dynamical system in discrete time with a transversal random homoclinic point.

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Cohomology of flows of stochastic and random differential equations

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Suppose a stochastic differential equation in $d-$dimensional Euclidean space possesses a global forward flow. We investigate conditions under which this flow is conjugate to the flow of a non-autonomous random ordinary differential equation, i.e. one can be transformed into the other via a random diffeomorphism of $d-$dimensional space. Viewing a stochastic differential equation in this form which appears closer to the setting of ergodic theory, can be an advantage when dealing with asymptotic properties of the system.

Lyapunov's Second Method for Stochastic Differential Equations

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During the past century Lyapunov's second method has gained increasing significance and has given decisive impetus for modern development of stability theory of dynamic systems. A manifest advantage of this method is that it does not require the knowledge of solutions of equations and thus has exhibited a great power in applications. On the other hand, since Itô introduced his stochastic calculus about 50 years ago, the theory of stochastic differential equations has been developed very quickly. Naturally, Lyapunov's second method has been developed to deal with stochastic stability by many authors (cf. [1–11]). There are several books available expounding the main ideas of Lyapunov's second method for stochastic differential equations e.g. Arnold [1], Has'minskii [5], Kushner [7]. In recent years, the original ideas have been refined, generalized and extended in several directions. Many new concepts have been introduced. Especially, some new ideas and approaches which might provide an exciting prospect of further advancement are still in the initial stages of investigation. In this talk, some of these aspects will be discussed.

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The Conley Index for Random Dynamical Systems

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The Conley index provides a topological tool in the qualitative study of dynamical systems. We will extend this method to random dynamical systems, which serve as a model for dynamics influenced or perturbed by probabilistic noise.

Criteria for the existence of random invariant sets and invariant measures under the presence of noise are given. In particular we will present some results on stochastic stability of invariant objects.

On the rate of dispersion of stochastic flows

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Suppose that $\phi_t(x)$, $x \in \mathbb{R}^d$, $t \geq 0$ is a stochastic flow of homeomorphisms on $\mathbb{R}^d$. We study the growth of the diameter of the image $\phi_t(B)$ of the unit ball $B \subset \mathbb{R}^d$ as $t \to \infty$. Using *chaining* we show that under suitable Lipschitz and boundedness conditions on the local characteristics of the flow, $\phi_t(B)$ grows at most linearly in t. In the second part of the talk we discuss the special case of an *isotropic Brownian* flow in more detail. We show that if the top Lyapunov exponent is nonnegative, then the diameter of $\phi_t(B)$ (and even that of $\phi_t(S)$ for any nontrivial bounded connected subset S of $\mathbb{R}^d$) actually grows linearly in t i.e. $\liminf \frac{1}{t} diam\,(\phi_t(B)) > 0$ almost surely. If – on the other hand – the top Lyapunov exponent is negative, then the following dichotomy holds: with probability one $\phi_t(B)$ either grows linearly or shrinks to a point as $t \to \infty$ and both events happen with strictly positive probability.

The talk is based on joint work with M. Cranston (Rochester) and D. Steinsaltz (Berkeley). The problem is motivated by applications in oceanography (the temporal dispersion of an oil slick on the surface of the ocean).

Contributed talks

The comparison of the stability of two statistical parameter estimators under complex disturbances in the data

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In the report, we make a comparison of the orthogonal regression (OR) estimator and the least squares (LS) estimator of the parameters of the autoregressive system:

$$x_t^* = \sum_{i=1}^{n} a_i x_{t-i}^* + \sum_{i=0}^{n} b_i u_{t-i}^*, \quad t = n+1, \ldots, N, \quad (N > n+1).$$

The both "proper" and "non-proper" disturbances are considered, namely, those under which the estimators do not possess the estimate consistency, asymptotic unbiasedness, etc. We prove the theorem about the LS and the OR estimators identity as functions of data in the first order term of the Taylor-series expansion with respect to the "true" non-perturbed data value. Then, by Monte-Carlo analysis, we investigate the region of the estimates' linear dependence on the data under small disturbances. By the above theorem, in this region the LS and the OR estimators are identical. To implement more thorough Monte-Carlo simulations, the terms up to third order in the Taylor-series expansions for both the LS and the OR estimation criteria were analytically achieved. It was found that under circumstances considered as quite general, with non-proper disturbances, the OR estimator yields less mean square deviation, than the LS estimator.

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New Aspects of Non-Integrability of Multi-Dimensional Systems Based on Symbolic Dynamics

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Some essentially new approaches and results in the problem of analytic non-integrability of multi-dimensional systems are presented. Traditionally, the analytic non-integrability means the absence of non-trivial analytic (or even meromorphic) first integrals, i.e., functions constant along each trajectory of the system under consideration. The problem of obtaining conditions that guarantee these non-integrability properties is the classical one and goes back to H. Poincaré. Results related to lower-dimensional systems (two-dimensional mappings or three-dimensional flows, in particular, Hamiltonian systems with two degrees of freedom) are well-known. Nevertheless, the number of rigorous results related to multi-dimensional systems is very limited.

We will describe and discuss conditions that guarantee the absence of any meromorphic first integral in the multi-dimensional case, regardless of a finite dimension in question. This is the strongest analytic non-integrability and not any Hamiltonian nature (which is of no importance) is assumed here. The conditions obtained persist under small perturbations and can be constructively verified for concrete dynamical systems in mechanics and physics. Some relevant results were published in [1,2].

Firstly, it is well-known that under some simple conditions the presence of transversally intersecting separatrices of hyperbolic periodic solutions implies the existence of a set of the so-called "quasi-random" motions. These motions are described by methods of symbolic dynamics. In the case of a two-dimensional mapping or a three-dimensional flow, the presence of quasi-random motions leads to the analytic non-integrability. The corresponding proof by V. M. Alekseev is not already valid for higher dimensions. Specific multi-dimensional conditions will be described that guarantee the non-integrability. The cause of the latter phenomenon is that the set of quasi-random motions is "rough enough" and does not lie on any smooth regular submanifold of positive codimension.

Secondly, we consider branching of solutions in the complex domain from the point of view of symbolic dynamics methods. This allows us to reformulate the above non-integrability conditions to the present case. The conditions obtained are easily applicable for a wide class of dynamical systems arising in mechanics and physics, in particular, those where other non-integrability approaches are not applicable or very hardly applicable. Even in the lower-dimensional case, the conditions obtained are simpler and more general than the known ones.

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Lyapunov Exponents of Linear Stochastic Jump Diffusions

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Under the uniformly elliptic condition or the hypoelliptic condition as in the diffusion case, see [1,2], the linear stochastic jump diffusion process projected on the unit sphere is a Fellor process and has a unique invariant measure using the relation of the transition probability between diffusions and jump diffusions in [3]. The deterministic Lyapunov exponent has an integral expression on the sphere with respect to the ergodic invariant measure so that the almost sure exponential stability of the linear stochastic jump systems depends on the its sign. The critical case of zero Lyapunov exponent will be discussed and examples will be treated as appropriated.

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Unified Conditions for Parameter Identifiability in AR, ARMA, ARMAX, EIV stochastic linear systems

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We consider the class of parameterized stochastic systems which includes AR, ARMA, ARMAX, EIV, and AR(I)MAX model structures. Data samples are supposed to consist of records of finite length, with the sample size depending on the number of records. It is shown that under natural assumptions about distributions of initial values, input variables, and disturbances the systems have parameter identifiability conditions expressible by restrictions on ranks of the real matrices of the system coefficients. For the "proper" parameterizations, derived conditions are not only sufficient but necessary. The results remain valid for the systems with data formed by samples of continuous records with a length tending to infinity.

The system is described by the equations

$$[\gamma_0(\theta) + \cdots + \gamma_p(\theta)s^p]\,\tilde{w}(t) = [\mu_0(\theta) + \cdots + \mu_p(\theta)s^p]\,\varepsilon_1(t)$$
$$w(t) = \tilde{w}(t) + \varepsilon_2(t), \qquad t \in \{1, \ldots, N-p\}.$$

Input-output signals are joined together in the vector variable $\tilde{w}(\cdot) \in R^{(r+m)}$, s is a backward shift operator, $(s\tilde{w}(t) \doteq \tilde{w}(t+1))$, and $\varepsilon_1(\cdot)$, $\varepsilon_2(\cdot)$ are unmeasured disturbances. Then, $\gamma_i(\theta) \in R^{r\times(r+m)}$, $\mu_i(\theta) \in R^{r\times l}$ are parameterized matrix coefficients of the system, and $\theta \in \Omega \subset R^v$ is a fixed parameter vector. The system does not undergo the controllability and stability restrictions.

Let us introduce *the measured trajectory* $z \doteq [w(1);\ldots;w(N)] \in R^{N(r+m)}$. In the paper, we investigate identifiability of the parameter value θ given the set $\{z_1,\ldots,z_L\}$ of measured trajectories. Consistency of the θ estimates is considered in the limit $L \to \infty$.

Let $\mathbf{P}_\theta$ be the distribution of z caused by given distributions of $\varepsilon_1(\cdot)$, $\varepsilon_2(\cdot)$, $u(\cdot)$, and w_0. The system is called *globally identifiable at* θ, if $\mathbf{P}_\theta = \mathbf{P}_\xi$, $\xi \in \Omega$ implies $\xi = \theta$. The system is called *locally identifiable at* θ, if there exists a neighborhood $V(\theta) \subset \Omega$ such that $\mathbf{P}_\theta = \mathbf{P}_\xi$, $\xi \in V(\theta)$ implies $\xi = \theta$ [1].

The problem statement: *It is needed to define restrictions on the parameterization $\theta \leftrightarrow \varphi_\theta$, under which the system is locally (globally) identifiable at $\theta \in \Omega$.*

Theorem. *The system is globally (locally) identifiable at θ, if and only if the equation*

$$\rho(s)\,[\gamma_\theta(s), \mu_\theta(s)q] = [\gamma_\xi(s), \mu_\xi(s)]$$

with respect to unimodular $\rho(s) \in R^{r\times r}[s]$, orthogonal $q \in R^{l\times l}$, and $\xi \in \Omega$ ($\xi \in V(\theta) \subset \Omega$), implies $\rho(s) \equiv I$, $q = I$, $\xi = \theta$.

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1.5 Symmetry

Organizers : Michael J. Field, Martin Golubitsky

Key note lecture

Bifurcation with Euclidean symmetry:
Universality, Reduction and Ginzburg-Landau equations

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I will consider mathematical issues concerning Ginzburg-Landau theory, by which I mean the validity, universality, and structure, of reduced equations near criticality in spatially extended systems. The extraction of Ginzburg-Landau equations (variously known as amplitude, modulation and envelope equations) is part of this theory. Particular attention is paid to the Euclidean symmetries present in such systems.

In local bifurcation theory, the word 'universality' has a precise meaning and refers to those aspects of local bifurcations that are driven by the type of bifurcation and the symmetries that are present. This point of view goes back to Landau's theory of second order phase transition in crystals, and is made mathematically precise for general problems with compact symmetry group in equivariant bifurcation theory [1]. Spatially extended systems have noncompact, usually Euclidean, symmetry, and the Ginzburg-Landau equations are supposed to be universal (in the above sense) for bifurcations in such systems. In my talk, I will describe complete results on universality for local bifurcations in systems with Euclidean symmetry. Some of these results can be found in [2] and show that the theory is richer than is often realized.

Another feature of Landau theory and equivariant bifurcation theory is reduction, locally, to a minimal system of reduced equations (for example, by center manifold reduction). I will present such a reduction method, preserving 'essential' solutions, for systems with Euclidean symmetry. Again, some of these results can be found in [2].

The extraction of Ginzburg-Landau equations corresponds to truncation of the reduced equations at lowest order. This step is problematic even formally. In situations where the extraction of Ginzburg-Landau equations is possible, their (exact, for all time) validity is clarified by the methods described in this talk. In addition, the presence and implications of normal form symmetry in the Ginzburg-Landau equations will be discussed.

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Invited lectures

Drifts for Euclidean Extensions of Dynamical Systems

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(This is joint work with I. Melbourne and M. Nicol.) We consider the behaviour of generic group extensions of ergodic dynamical systems by noncompact groups, in particular by $SE(n)$ the special euclidean group. A natural question of physical interest is whether the norms of solutions of such systems are unbounded or not, and if they are unbounded then at what rate do they typically grow. For extensions of uniformly hyperbolic mappings we show using a central limit theorem argument that there are generically unbounded orbits whose variance grows proportional to time. For group extensions of quasiperiodic and periodic we give conditions that ensure generic boundedness and unboundedness. Some implications of these results on the behaviour of instabilities of reaction diffusion systems on unbounded domains is also considered.

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Bifurcation of relative equilibria in Hamiltonian systems

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The relative equilibria of a symmetric Hamiltonian dynamical system are the critical points of the so-called *augmented Hamiltonian.* In the same spirit as in the work of Krupa [1] in the context of non-conservative systems, the underlying geometric structure of the system is used to decompose the critical point equations and construct a collection of implicitly defined functions and reduced equations describing the set of relative equilibria in a neighborhood of a given relative equilibrium. The analytical and group invariance structure of these reduced equations is exploited to solve the problem in specific cases. In particular, a persistence result of Lerman and Singer [2] is generalised to the framework of Abelian proper actions. The method is illustrated with an example from wave resonance in mechanical systems which was initially studied with a different method by Chossat and Dias [3].

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Stable ergodicity

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We describe recent work on stable ergodicity for diffeomorphisms equivariant with respect a compact Lie group. Some of our results are joint work with Parry (Warwick) [1] and Nicol (UMIST) [3]. One of the techniques we use depends on constructing a symbolic dynamics (finite coding) on the orbit space of a group invariant (partially) hyperbolic set. This is done using a generalization of Markov partitions appropriate for the study of non-free group actions. (In general, dynamics on the orbit space will not be expansive.)

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Animal Gaits and Coupled Oscillators

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The observed spatio-temporal symmetries of animal gaits — that is, the rhythmic patterns of movement of the legs — provide useful clues to CPG architecture. Plausible assumptions about the symmetries of a CPG for quadrupedal locomotion that generates the range of symmetries of observed gaits imply that such a CPG should consist of eight nominally identical subcircuits, or 'cells', arranged in an essentially unique manner. Two cells are assigned to each limb, and it is conjectured that their role is to control the timings of two distinct muscle groups. Analogous CPG architecture is proposed for bipeds, hexapods, and myriapods.

Predictions include the following. Gaits naturally divide into two classes: primary (walk, trot, ...) and secondary (gallops, canter, ...). There is a new primary gait called *jump*. There are restrictions on the wave numbers of primary gaits of myriapods of which the common tripod gait of hexapods is an example. Bipeds perform two distinct gaits with the 'out-of-phase' symmetry of the walk, and two distinct gaits with the 'in-phase' symmetry of the hop. Some evidence suggests that the two walk gaits in humans are the usual walk and run gaits; in the walk the limbs flex at the ankle, and in the run the ankle is more rigid.

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Forced symmetry breaking and pattern formation in weakly anisotropic systems

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Pattern formation in two-dimensional weakly anisotropic systems is considered from the point of view of forced symmetry breaking. In the absence of anisotropy the problem is formulated on an isotropic spatially periodic lattice with a symmetry group G. This group acts on the amplitudes $\mathbf{z}$ via its representation Γ [1]. In the presence of anisotropy the group G acts simultaneously on the (two-dimensional) applied anisotropy vector $\mathbf{s}$, but does so in general with a different representation Γ'. The notion of (Γ, Γ') co-equivariance is introduced to describe dynamical systems which are G-symmetric in the sense that

$$\gamma(g) \cdot \mathbf{f}(\mathbf{z}, \mathbf{s}) = \mathbf{f}(\gamma(g) \cdot \mathbf{z}, \gamma'(g) \cdot \mathbf{s}), \qquad \forall g \in G. \tag{1}$$

Using Poincaré series the most general (Γ, Γ') co-equivariant systems are constructed [2]. The resulting equations for $G = T^2 \dot{+} D_6 \oplus \mathbb{Z}_2$ and $G = T^2 \dot{+} D_6$, corresponding to pattern formation on a hexagonal lattice with and without midplane reflection symmetry, are truncated at third order in both $\mathbf{z}$ and $\mathbf{s}$ and analyzed. The results are applied to Rayleigh-Bénard convection in weak imposed Poiseuille and Couette flows.

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Bifurcation from discrete rotating waves

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My talk will address recent developments in the understanding of bifurcation from periodic solutions in equivariant dynamical systems. The focus will be mainly on the theory for bifurcation from *isolated* periodic solutions in dynamical systems with a compact symmetry group.

Equivariant bifurcation theory is concerned with the bifurcations of equilibria and periodic solutions in differential equations that are equivariant with respect to the action of a (compact) Lie group. A systematic approach to bifurcation from symmetric equilibrium solutions is laid out in [1]. Importantly, bifurcations in equivariant systems are generally different from bifurcations one would expect in non-symmetric systems.

Until recently, a theory for bifurcation from symmetric periodic solutions was developed only to deal with certain special situations. The theory for periodic solutions with purely spatial symmetries was developed by Chossat and Golubitsky [2]. Fiedler [3] studied systematically bifurcation from periodic solutions with discrete spatiotemporal symmetry using return map techniques. However, the latter study was confined to cyclic symmetry groups.

The treatment of bifurcation from periodic solutions with spatiotemporal symmetries using return map techniques was taken up again recently by Lamb [4]. It turns out that an extension of Fiedler's approach involves consideration of *twisted equivariant* maps (called k-symmetric maps in [4,6]). In [5], Lamb and Melbourne developed a systematic theory for generic bifurcation from spatiotemporally symmetric periodic solutions (with compact symmetry groups) that are isolated in phase space, using the return map approach.

My talk will survey some of the main principles and results in studying bifurcation from periodic solutions with spatiotemporal symmetry.

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Translational Symmetry-Breaking for Meandering Spirals

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Mathematical models with Euclidean symmetry successfully describe the meandering behavior of spiral waves (e.g. in Belousov-Zhabotinsky chemical reactions). However, no physical experiment is ever infinite in spatial extent, so the translational symmetry is only approximate. As with any approximation, it is natural to question the circumstances and the extent to which it is valid. For example, in the Belousov-Zhabotinsky reaction, the Euclidean model is extremely successful. However, for spiral waves in cardiac tissue, one expects the presence of blood vessels and other inhomogeneities to render this Euclidean approximation less than ideal. There is thus a range of problems involving spiral waves which are more or less well-modeled with full Euclidean symmetry. It would therefore be interesting to characterize the effects of system symmetry-breaking terms on the meandering behavior of spirals.

In this talk, we investigate the effects on spiral wave dynamics of breaking the translation symmetry while keeping the rotation symmetry. This is accomplished by introducing a perturbation in the five-dimensional center bundle equations (describing Hopf bifurcation from one-armed spiral waves) which is SO(2)-equivariant but not equivariant under translations. This system symmetry-breaking term induces a “bifurcation at infinity” which is analyzed in detail.

Time-Reversible Equivariant Linear Systems

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This talk will describe joint work with Igor Hoveijn (Groningen) and Jeroen Lamb (Warwick/Imperial) on the normal form theory of linear systems of differential equations which are equivariant with respect to actions of half the elements of a compact Lie group and time-reversible with respect to the other half. General and Hamiltonian linear systems will both be considered. At the heart of the theory lies a representation theory result which reduces the classification of all such systems to the study of 10 ‘universality classes’. For some of these universality classes the normal form theories are isomorphic to Jordan normal form theories of classical Lie algebras, but for the others new classifications are needed.

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Bifurcations from relative periodic solutions

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Relative periodic solutions are ubiquitous in dynamical systems with continuous symmetry. Recently, Sandstede, Scheel and Wulff derived a center bundle theorem, reducing local bifurcation from relative periodic solutions to a finite dimensional problem. Independently, Lamb and Melbourne showed how to systematically study local bifurcation from isolated periodic solutions with discrete spatiotemporal symmetries.

In this talk, we show how the center bundle theorem, when combined with certain group theoretic results, reduces bifurcation from relative periodic solutions to bifurcation from isolated periodic solutions. In this way, we obtain a systematic approach to the study of local bifurcation from relative periodic solutions.

Contributed talks

Symmetries in Exterior Differential Systems

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Using Lie's symmetry approach for integrating Frobenius integrable vector field distributions, we present two symmetry techniques for generating solutions of first order nonlinear PDEs. In the first, we use the idea of Cauchy characteristics to show how one is able to solve the Cauchy problem using symmetries.

In the second, we consider a first order non-linear PDE and derive its corresponding *Vessiot distribution*, whose integrability conditions generate a first order quasilinear PDE. We then show that Lie's symmetry approach can be used to easily find a family of solutions of this quasilinear PDE that can then be used to generate solutions of the original first order non-linear PDE.

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Hopf bifurcations on the FCC lattice

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When a system undergoes a Hopf bifurcation to a pattern forming instability with spatial symmetry group Γ, the Equivariant Hopf Theorem [1] guarantees a unique primary solution branch for every $\mathbb{C}$-axial isotropy subgroup of $\Gamma \times S^1$, where S^1 represents the group of temporal phase shifts. For Γ a wreath product group, the method of Dias [2] can be used to determine all such $\mathbb{C}$-axial subgroups easily. In this paper we analyze three-dimensional pattern forming Hopf bifurcations with the spatial periodicity of the face-centered cubic (FCC) lattice. This is an equivariant Hopf bifurcation with spatial symmetry $\Gamma = T^3 \dot{+} \mathbb{O} \oplus \mathbb{Z}_2$, where T^3 is the 3-torus of translations, $\mathbb{O}$ is a representation of the octahedral group of orientation-preserving symmetries of the cube, and the nontrivial element of $\mathbb{Z}_2$ represents spatial inversion through the origin. We use Dias' method by first extending Γ to the larger group $O(2) \wr S_4$. By relating the two sets of $\mathbb{C}$-axial subgroups, we find all solution branches guaranteed by group theory to be primary. We use the Poincaré series to find the most general possible equivariant system of amplitude equations up to cubic order, which determines the possible bifurcation diagrams for the primary solution branches. Although motivated by reaction–diffusion problems, our results are done in a model-independent fashion, and are applicable to a wide variety of pattern formation problems near onset. This work is an extension of that done for the steady state FCC bifurcations in [3, 4].

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The relations between Painlevé property and Lie symmetries of differential equations

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Second order ordinary differential equations can be classified by investigating the realizations of real low dimensional Lie algebras in terms of vector fields in two dimensions [1]. An equation admitting r-dimensional, r=1, 2, 3 or 8, Lie algebra of point symmetry generators possess a canonical representative for the corresponding differential equation.

For the case $r = 1$ the differential equation can be transformed to an autonomous form. The case $r = 2$ gives two classes of representative equations. For equations having three point symmetries, there are five representatives of equivalance classes. For $r = 8$, a number of linearizability results proved in literature [1].

It is of great interest to compare the Lie classification of second order equations with the Painlevé classification which produces 50 equations [2,3]. The question which arises is this: Is there an overlap between the Lie classification and the Painlevé classification? The first step in answering this question is to determine the symmetries of the Painlevé equations. Alternatively one has to reduce some of the Lie equations to the Painlevé ones.

The aim of this work is to bring some light on this issue, using restricted point transformations.

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Polynomial Infinitesimal Generators of Lie's Symmetries for Polynomial Planar Vector Fields

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In this work we show that if all the intersection points of the projective curves defined by the components of the polynomial vector field submerged in $\mathbb{CP}^2$ are different, the critical points of the components of the polynomial planar vector field are strong and there exists a polynomial inverse integrating factor then there exists a polynomial infinitesimal generator of Lie's symmetry. The result continues being valid if all the critical points, be strong or not, belong to the algebraic curve defined by the polynomial inverse integrating factor. In these cases the Lie's symmetries are related with the class of funtions which belongs to the inverse integrating factor independently of the class of their first integral.

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Generic bifurcation from a 2-manifold

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Systems with continuous symmetry typically have equilibrium sets that are n-manifolds with $n > 0$. Under symmetry-breaking perturbations such a manifold breaks up into lower-dimensional pieces or to a discrete set of points. We describe the geometry of equilibrium configurations that arise generically in the particularly interesting case when $n = 2$ (for spherical or toroidal symmetry, for example), both when the symmetry is totally broken and when some residual symmetry persists.

On the stability of bifurcating periodic orbits in reversible and symplectic mappings

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We survey some recent results on the bifurcation of periodic points from a fixed point in parametrized families of reversible or symplectic diffeomorphisms; such bifurcation problems arise for example when studying subharmonic branching in reversible or Hamiltonian systems. In some joint works with A. Vanderbauwhede we analyzed some elementary "branching phenomena" which can occur near a fixed point; we provided a "structure-preserving" reduction result which can be used to study such branchings and we also briefly discussed how one can determine the stability of bifurcating periodic orbits. Here, we will fix our attention on this last problem, pointing out the primary role played by normal form theory. As an application we provide the complete analysis of the stability of periodic orbits appearing in the simplest bifurcation scenario for both Hamiltonian and reversible systems.

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Smooth normalization of reversible vector fields

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It is shown that Sternberg linearization theory remains true in the category of reversible vector fields.

Let $R : \mathbb{R}^n \to \mathbb{R}^n$ be a linear involution, *i.e.*, $R^2 = I$. A vector field ξ is said to be *R-reversible* if

$$R_* \xi = -\xi.$$

Two vector fields ξ and η are called *R-equivalent* if there is a diffeomorphism $H : \mathbb{R}^n \to \mathbb{R}^n$ such that $H \circ R = R \circ H$ and

$$H_* \xi = \eta.$$

Theorem. *Let k be a positive integer and A be an R-reversible hyperbolic linear vector field. There exists a number $Q = Q(A, k)$ such that if two smooth vector fields ξ and η are R-reversible, $\xi(0) = \eta(0) = 0$, $A \equiv D\xi(0) = D\eta(0)$, and the vector field $(\xi - \eta)$ is Q-flat at the origin, then ξ and η are locally C^k R-equivalent.*

This solves the problem recently posed by Bonckaert.

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Calculation of the solution of Boussinesq Problem using Lie symmetries

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Firstly, the Lie point symmetries of cylindrically symmetric homogeneous Navier equations are obtained. Using the symmetries the general class of similarity solutions is found. The subclass that also satisfies the non-homogeneous system of the medium subject to a singular force is determined. Substituting the subclass into the non-homogeneous system, a system of ordinary differential equations is obtained. The solution of the system satisfying the boundary conditions of Bousssinesq problem gives the exact solution.

Applications of a new G-Invariant Implicit Function Theorem

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The presentation concerns a generalization [1, Theorem 3.1] of the G-invariant implicit function theorem of E. Dancer [2]. It will be shown how this generalization can be applied to rotating and modulated wave solutions of nonlinear equivariant autonomous or periodic evolution equations (with applications to laser dynamics) and to nonlinear elliptic boundary value problems on bounded symmetric domains (with applications to elastostatics).

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On time-dependent symmetries of evolution equations

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We investigate the symmetry algebra of the generic scalar (1+1)-dimensional evolution equation. Using some auxiliary results, we have found the formulae for the leading terms of commutators of two symmetries and two formal symmetries and have proved several general theorems on the structure of the algebra in question. The generalization of the above results to the case of system of evolution equations is also discussed.

References

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Hopf-zero Bifurcations of Reversible Vector Fields

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Let X be a (germ of) C^∞ vector field defined on $\mathbb{R}^n, 0$ given by an ordinary differential equation

$$X: \quad \dot{x} = F(x), \quad x \in \mathbb{R}^n, \tag{1}$$

where $F(x)$ is a smooth function, $F(0) = 0$. We say that system (1) is time-reversible if there is a germ of a smooth involution $\phi: \mathbb{R}^n, 0 \to \mathbb{R}^n, 0$ ($\phi \circ \phi = id.$) such that the relation

$$F(\phi(x)) = -\phi'(x) \cdot F(x), \quad x \in \mathbb{R}^n, 0 \tag{2}$$

holds.

A φ time-reversible vector field X on $\mathbb{R}^n$, 0 is called to be of type (n, k) if the dimension of the fixed point set of φ, $S = Fix\{\phi\}$, is equal to k.

The main objective of the paper is to study symmetric critical points of generic one-parameter families of reversible vector fields of type $(3, 1)$ with emphasis on the topological classification of all such systems. We assume that all the systems considered here are ϕ_0 reversible, where $\phi_0(x, y, z) = (-x, -y, z)$. Note that in generic case the eigenvalues of such systems are either $(0, \alpha, -\alpha)$ or $(0, \alpha i, -\alpha i)$, where α is a nonzero real parameter. We focus on systems which have eigenvalues $(0, \alpha i, -\alpha i)$.

The main results of this paper include a classification and qualitative behavior of symmetric singularities occurring generically for 1-parameter families of (3,1)-type vector fields. Also interesting conclusions are obtained from the classification which involves the existence of invariant tori, periodic orbits, homoclinic and heteroclinic orbits for vector fields considered. We also give a complete list of bifurcation diagrams.

On symmetry reduction in reversible and Hamiltonian systems

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We consider reversible systems which are equivariant with respect to a compact continuous group of symmetry operators compatible with the reversibility. We use the approach of M. Krupa to study bifurcations near relative equilibria which are invariant under the reversing symmetry. We show that such bifurcations are completely determined by the bifurcations near an equilibrium of a reversible "normal" vectorfield. We discuss several examples in detail. We illustrate a similar approach for Hamiltonian systems on an example, namely when the symmetry group is S^1. For this Hamiltonian case a general theory along these lines is still lacking, but we hope to present at least some new partial results.

Symmetries and Mode Interactions in the Optimisation of Hydrophone Placement in Acoustic Arrays

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The placement of a number of hydrophones (underwater microphones) in an array is studied with a view to optimising the signal to noise ratio. This leads to an optimisation problem for a performance measure on $\mathbb{R}^{2n}$ for the horizontal array case (where we are optimising over the spatial configuration of n hydrophones). That is, the position of the ith hydrophone is given by $x_i \in \mathbb{R}^2$ and the array configuration is given by $\underline{x} = (x_1, \ldots, x_n)$. The performance measure derived in Haywood (1994) is of the form

$$\nu(\underline{x}) = \frac{1}{k_2 - k_1} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{W(k_2|x_i - x_j|) - W(k_1|x_i - x_j|)}{|x_i - x_j|}.$$

Here $k_i = 2\pi f_i/c$, where $f_1 \leq f \leq f_2$ is the frequency range which the array is designed to search, and the function W is defined as

$$W(u) = \int_0^u J_0(v)^2 dv$$

where J_0 is the Bessel function of order 0.

Both this underlying performance measure, and the solutions observed by simulated annealing (Haywood 1994), exhibit strong symmetry properties. In particular, optimisation of the performance measure can be reduced to considering a set of equations which are $O(2) \times S_n$ equivariant. These equations can be augmented with a parameter λ which allows us to consider a bifurcation from the trivial solution of all the hydrophones lying at the same co-ordinate. By considering the solutions branches we would expect to see in such a system, including simple mode-interactions (using the standard approach of, for example, Golubitsky et. al. 1988) we reduce the problem to optimising a series of functions living on $\mathbb{R}$ and $\mathbb{R}^2$ which provides a systematic sweep across many of the local minima (for a array with n hydrophones we require 2 optimisations of one-dimensional functions and $n/2$ or $(n-1)/2$ optimisations over $\mathbb{R}^2$ depending on whether n is even or odd).

At the very least the results thus obtained matches the results obtained through simulated annealing for finding global minima using an extremely cheap numerical method, and in addition provide a useful classification scheme for possible configurations.

References

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1.6 Topological Methods and Conley Index

Organizers : Konstantin Mischaikow, Marian Mrozek

Key note lecture

On the Conley index over a base space

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Basing on the papers [1] and [2], we present a recent generalization of the Conley index theory. Let S be an isolated invariant set of a flow ϕ on a locally compact metrizable space X. For a given continuous map $\omega : X \to Z$, where Z is a Hausdorff space, we define the Conley index of S over the base space Z as the so-called fiberwise deforming homotopy type of the adjunction of an isolating block B for S and Z, via the map $\omega|_{B^-} : B^- \to Z$, where B^- is the exit set of B. Usually the index over a base provides more information then the classical Conley index if X is not contractible. The additional information can be applied to problems on continuation of isolating invariant sets. In the case of the suspension flow of a homeomorphism and Z equal to the circle, the new index can replace the discrete-time Conley index.

References

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Invited lectures

On transition matrices

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From the Conley index theory, a transition matrix gives algebraic information about the existence of codimension one connecting orbits between Morse components of a Morse decomposition of an isolated invariant set in a one parameter family of flows. There are basically three different formulations of transition matrices: singular, topological and algebraic transition matrices. In this talk, I shall discuss relations of these formulations and attempt to put them in a unified framework. As an outcome of this new view, I present a generalization of transition matrices for a family of flows with more than one parameters.

Periodic orbits for fourth order conservative systems and a Morse type theory

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For a large class of fourth order Lagrangian systems we can find periodic orbits on prescribed energy manifolds via analogues of area preserving twist-maps. Appropriate generating functions give rise to second order recurrence relations. By studying the associated gradient flow we find a multitude of periodic orbits (infinitely many in most cases). The arguments are based on a Morse type theory via the Conley index, where we use 'braid types' of periodic orbits to define isolating neighborhoods. The results apply for example to the Swift-Hohenberg and the extended Fisher-Kolmogorov equations [1], [2]. This is joint work with J.B. Van den Berg.

References

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Contributed talks

The implementation of the Allili-Kaczynski algorithm of the construction of chain homomorphism induced by multivalued map

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Our talk presents an implementation of the algorithm for computing chain homomorphisms induced by multivalued representable maps. For simplicity and efficiency of data representation, we use the cubic grid, which does not require computing of subsequent barycentic subdivisions. The theoretical background for the algorithm comes from Allili and Kaczynski. This is a part of a joint project of computing the Conley Index for discrete dynamical systems.

References

[1] M. Allili, T. Kaczynski. An algorithmic approach to the construction of homomorphisms induced by maps in homology. preprint.

Computer Assisted Proof of the Existence of a Periodic Orbit in the Rössler Equations

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In 1976, Rössler [4] introduced an example of a set of nonlinear differential equations in $\mathbb{R}^3$: $x' = -(y+z)$, $y' = x + by$ and $z' = b + z(x-a)$. Its numerical simulations show a variety of qualitative behaviours as the parameter a is changed, to begin with a stable fixed point through an attracting periodic orbit which experiences a series of period-doubling bifurcations to finally reach a strange attractor. Indeed, for $a = 5.7$ and $b = 0.2$ the existence of chaotic dynamics has been recently proved by Zgliczyński [5].

The values of the parameters discussed here are $a = 2.2$ and $b = 0.2$. For these values an attracting periodic orbit is numerically observed, but as far as we know there has been no strict proof of this fact so far. Our aim is to fill this gap.

In this proof of the existence of the periodic orbit, computer was used to construct heuristically a set N built of a huge number of tiny cubes to be a candidate for an isolating neighbourhood of the hypothetic orbit. For arbitrarily chosen $t > 0$, Lohner's method for strict integration of ordinary differential equations was used to prove that the image of N with respect to the time-t map was entirely included in the interior of N. In such a way an index pair $(N, \emptyset)$ was obtained. Then an algorithm by Kaczyński, Mrozek and Ślusarek [1] was used to compute homology of N, which was the homology of a circle S^1 as expected. According to a theorem by Mrozek [3], the maximal invariant set S contained in N with respect to the discretisation of the flow was also an isolated invariant set for the flow itself. Moreover, it had the same Conley Index. After some more computations performed in order to check additional assumptions, a theorem by McCord, Mischaikow and Mrozek [2] was applied to finally obtain the existence of a periodic orbit in this neighbourhood.

The method used here may make use of other algorithms being recently implemented to compute a discrete Conley Index of an isolating neighbourhood and to use the same theoretical basis to prove the existence of a periodic orbit even if it is not necessarily attracting. Moreover, this method is dimension-independent and thus may be used to prove the existence of a periodic orbit in $\mathbb{R}^n$ also for $n \neq 3$.

References

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Sensitive dependence on initial conditions using topological tools ?

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In my talk I would like to address the following question: *Can we say something about sensitive dependence on initial conditions, using topological tools only ?*

At first sight the answer to this question is negative, because topological tools like Conley index or fixed point index are tools which do not see what happens in very small portions of the phase space. The silent assumption behind this reasoning is that we use only finite number of sets on which we compute various topological invariants.

In this talk we discuss how to show existence of complicated orbits which are homoclinic to a hyperbolic periodic orbits both in discrete (map) and continuous (ODE) case.

Discrete case: Let x_0 be a hyperbolic fixed point for a homeomorphism $f : \mathbb{R}^n \to \mathbb{R}^n$. We can then construct a sequence of parallelograms N_i, $i \in \mathbb{N} \cup \{0\}$, $x_0 \in N_i$

$$N_{i+1} \subset N_i, \quad \lim_{i->\infty} \mathrm{diam}(N_i) = 0, \quad N_{i+1} \overset{f}{\Longrightarrow} N_i, \quad N_i \overset{f}{\Longrightarrow} N_{i+1}$$

where the symbol $\Longrightarrow$ denote a covering relation introduced in [1].

Suppose now that we have a loop of covering relations

$$N_0 \overset{f}{\Longrightarrow} M_1 \overset{f}{\Longrightarrow} M_2 \overset{f}{\Longrightarrow} M_2 \ldots \overset{f}{\Longrightarrow} M_k \overset{f}{\Longrightarrow} N_0 \tag{1}$$

From this we conclude that the exists a point $x_0 \in N_0$ such that $\lim_{i \to \pm\infty} f^i(x) = x_0$ and $f^i(x) \in M_i$ for $i = 1, \ldots, k$, which can correspond to a possibly complicated behavior.

To illustrate the **continuous case** we consider a planar $T = 2\pi/\kappa$-periodic equation

$$z' = (1 + e^{i\kappa t}|z|^2)\overline{z}, \quad z \in \mathbb{C} \tag{2}$$

for $0 < \kappa \leq 0.495$. Here origin is a hyperbolic periodic trajectory γ of period T. In [2] it was shown using two isolating segments U, W that this equation has symbolic dynamics on three symbols, i.e. we have a complicated dynamics. This coresponds to the loop covering relations (1). We construct an isolating segment Z for γ, such that diameter of $\lim_{|t| \to \infty} \mathrm{diam}(Z_t) = 0$ and for some t $U_t = Z_t$. From this we obtain homoclinic orbits to 0 with a complicated dynamics (see [3] for details).

References

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