B. Infinite-dimensional Systems

2.1 Delay Equations

Organizers : Sjoerd Verduyn Lunel, Hans-Otto Walther

Key note lecture

Effects of time delays on the dynamics of feedback systems

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Recently there is a lot of attention devoted to feedback systems, i.e., optical feedback lasers, phase-locked frequency synthesizers and wave equations with feedback stabilization at the boundary. In the implementation of a feedback system, it is very likely that time delays will occur. It is therefore of vital importance to understand the sensitivity and robustness of the feedback system with respect to time delays in the loop. In this lecture we discuss a number of illustrative examples and some useful tools, based on the radius of the essential spectrum of the semigroup associated with the system, to study robustness with respect to time delays.

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Invited lectures

Chaotic motion in delay equations

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Since the late 1970s, chaotic dynamics in delay equations was observed numerically, and analytical proofs for the existence of such behavior in specific examples were obtained from 1980 on. We give a review of results in this direction up to the present, describing different mechanisms for the creation of erratic motion. Such mechanisms are: Conjugacy to chaotic interval maps, unstable periodic solutions and transversally homoclinic orbits, and Šil'nikov-like saddle connections.

The Global Dynamics of Cyclic Feedback Systems with Delay

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We show that in a cyclic feedback system with delay, no solution which has a finite rate of oscillation (as measured by a discrete Lyapunov function) can approach an equilibrium at a rate faster than exponential. As a consequence of this rather technical result, we obtain information about a variety of dynamical phenomena, including Floquet exponents and subspaces, Morse decompositions, and Morse-Smale properties (transversality) of such systems.

Synchronization and Phase-locked Oscillations in Delayed Neural Networks: Connecting Orbits and Basins of Attraction

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We consider the following system of delay differential equations

$$\begin{aligned}\dot{x}_1(t) &= -\mu x_1(t) + f(x_2(t-\tau)),\\ \dot{x}_2(t) &= -\mu x_2(t) + f(x_1(t-\tau))\end{aligned}$$

which describes the computational performance of a network of two identical neurons with delayed feedback, where x denotes the activation, $\mu > 0$ is the decay rate, $f : \mathbb{R} \to \mathbb{R}$ is the signal function and $\tau > 0$ is the synsptic delay. In many applications such as content-addressable memory, it is very important to give a detailed description of the structures of the global attractor.

The synchronized activities ($x_1 = x_2 = y$) of the network are characterized by the scalar DDE $\dot{y}(t) = -\mu y(t) + f(y(t-\tau))$. The detailed fine structures of a basic attracting invariant set as a 3-dimensional smooth solid spindle inside the global attractor of the scalar equation were presented in the monograph of Krisztin, Walther and Wu [The Fields Inst. Monographs Series, Vol. 11, AMS, 1999], the connection of this spindle with the global attractor was discussed in Krisztin and Walther [preprint, 1999], the singularity of the spindle at the tips was described in Walther [preprint, 1999], and the topological and geometric structures of high dimensional invariant sets are being investigated in Krisztin and Wu [preprint, 1999].

Recent work of Chen and Wu [Advances in Differential Equations, to appear; Physics D, to appear] and Chen, Krisztin and Wu [J. Differential Equations, to appear] shows that the general geometric approach developed in the aforementioned monograph, a discrere Lyapunov function which counts sign changes of elements in the phase space and the Poincare-Bendixson theory developed in Mallet-Paret and Sell [J. Differential Equations, 125, 385–489, 1996] for cyclic systems of DDEs can be effectively applied to describe the dynamics of certain sets of asynchronous orbits. In particular, we obtain when τ passes the first Hopf bifurcation value τ_1 an analog of the above 3-dimensional solid spindle which is separated into the basins of attraction of the two tips by a smooth disk bounded by a phase-locked orbit. We also show that the unstable space of the monodromy operator of the phase-locked orbit is at least 3-dimensional, when τ passes the second Hopf bifurcation value τ_2 where a bifurcation of the aforementioned solid spindle consisting of synchronous orbits, and there are connecting orbits from the synchronous periodic orbit to the phase-locked orbit. This provides a mechanism for desynchronization.

Limiting profiles as square waves or pulses of the phase-locked and synchronized orbits as the delay approaches infinity are also investigated in some recent works of Chen and Wu [preprint, 1999].

Contributed talks

Analytic solutions to partial differential equations with deviating arguments

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We investigate analytic solutions to the following problem

$$D_t u(t,z) = f\left(t, z, \left\{(I^i D_z^j u)(\alpha^{(i,j)}(t,z))\right\}_{i,j=0}^{K}\right) ,$$

$$u(0,z) = \phi(z) , \qquad (t,z) \in G_\eta ,$$

on the Haare pyramid

$$G_\eta = \left\{(t,z) \in \mathbb{C}^2 : z \in \Omega,\ |t| \le \eta \min\left(\hat{d},\, \mathrm{dist}(z, \partial\Omega)\right)\right\}$$

and Ω is an open subset of the complex plane with nonempty boundary.

The symbols D_t, D_z mean the partial derivatives and

$$(Iw)(t,z) = \int_0^t w(s,z)ds$$

We consider existence and uniqueness of analytic solutions and continuous dependence on known functions with respect to some weighted norms in a space of analytic functions.

Computational Scheme of a center manifold for neutral functional differential equations

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This work addresses a computational algorithm of terms of a center manifold for neutral functional differential equations. The Bogdanov-Takens and the Hopf singularities are considered. Finally, as an illustration of our scheme, we give an example where the second term of a center manifold is determined explicitly.

Real-Analytic Solutions of Functional Equations

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We consider an equation

$$\phi(x) - A(x)\phi(Fx) = \gamma(x), \qquad x \in \mathbf{R}^n,$$

in real-analytic functions $\phi(x)$ with given real-analytic matrix function $A(x)$, vector function $\gamma(x)$ and mapping $F : \mathbf{R}^n \to \mathbf{R}^n$. The relations are studied between local and global solvability of the equation. In particular we state conditions which guarantee that the local solvability implies the global one.

On existence of periodic solutions for a linear difference system with continuous argument under periodic control

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The purpose of this paper is a condition for the system of difference equations with continuous argument under continuous and periodic control $u(t)$

$$x(t+1) = Ax(t) + u(t); \tag{1}$$

$$x(\tau) = \varphi(\tau), \quad \tau \in [0,1),$$

that must be fulfilled for existence of a periodic solution. Here $u(t)$ is a continuous and T-periodic by t n-vector-function, $t \in R^+$, A is a real nonsingular $n\times n$ matrix, $x(t)$ is a real phase vector, $\varphi(t)$ - is an initial n - vector-function. Solution of this problem is an initial step in stabilization of a periodic motion for the system (1). An extensive literature is devoted to the theory of such kind of systems, for example [1,2]. There has been much interest recently in the construction of general or special solutions for difference systems with continuous argument, see e.g. [3,4]. In particular, existence conditions for a periodic solutions with an integer period of the system (1) were established, for example in [5].

This paper is devoted to existence conditions for periodic solutions of (1) with arbitrary period (both commensurable with the step of the system and incommensurable). The general solution of the homogeneous system

$$x(t+1) = Ax(t), \tag{2}$$

corresponding to (1) is constructed. Existence conditions of a periodic solutions for (2) are developed. It was shown that the system (1) has a T-periodic solution if and only if matrix A has an eigenvalue equal to $2\pi ik/T$, and the initial function $\varphi(\tau)$ in accordance with T has a form: for an integer T - $\varphi(\tau)$ is arbitrary 1-periodic function; for a rational $T = k/m$ (where k and m are mutually disjoint integers) - $\varphi(\tau)$ is $1/m$ - periodic by τ function; for irrational T - φ is arbitrary constant. For the system (1) the sufficient conditions for existence of a periodic solution with integer, rational and irrational period are obtained. The theory of normal form by Poincare is used [6, chap. 5]. The problem of series' convergence arising from the small devisors problem is investigated on the base of [6, p. 108]. Illustration examples are given.

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Neutral Equations with Causal Operators

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The paper is concerned with existence results, both local and global, for functional equations of the form $(Vx)(t) = (Wx)(t)$, in which V and W stand for causal (nonanticipative) operators on various function spaces.

The case of functional differential equations of the for $(d/dt)(Vx)(t) = (Wx)(t)$ is also covered.

Methods of Nonlinear Analysis are used in proving the existence results. Many special types of equations are treated as particular cases of the general equations mentioned above.

Asymptotic properties of functional differential equations

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New results on boundedness/unboundedness of solutions and exponential stability of differential equations with delayed argument and integro-differential equations are proposed.

A) In previous studies in this theme stability of integro-differential equations was achieved through their "ordinary part". As to their integral part, it was required to be small enough. In our approach stability is achieved through the "integral parts". Tests for exponential stability even in the case of "non-stable ordinary part" are proposed.

B)Delay in the following differential equations of a second order

$$\ddot{u}(t) + p(t)u(t - h(t)) = 0 \tag{1}$$

destroys stability. It is shown that for non-decreasing positive bounded coefficient $p(t)$ convergence of the integral $\int_0^\infty h(t)$ is necessary and sufficient for boundedness of all solutions. The case when $p(t)$ tends to infinity has been also considered.

Invariant manifolds for RFDE under discretization

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In our previous work we showed that the unstable manifold around hyperbolic equilibrium of a scalar retarded functional differential equation (RFDE for short) is preserved under the Euler method. That is, if $\mathcal{W}^U$ and $\mathcal{W}^{U_h}$ denote local unstable manifolds of the original and the Euler discretized equation, respectively, then $\|g - g_h\|_{C^0} \to 0$ as $h \to 0$, where g and g_h are functions representing the manifolds locally as graphs and h is the step size.

The aim of the present talk is two-fold. On one hand we generalize the above mentioned result by allowing general RFDE and general numerical method. On the other hand we show the higher order closeness of invariant manifolds, i.e. $\|g - g_h\|_{C^j} \to 0$ as $h \to 0$, where j depends on the order of the numerical method. The proof is based on the Parameterized Contraction Mapping Principle while smoothness follows from the Fibrum Contraction Theorem.

Connections to the Hartman-Grobman Theorem are also discussed. Using it we obtain conjugacy between the solution flow and its numerical approximation on a neighborhood of the hyperbolic equilibrium lying on the global attractor.

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On error estimates for waveform relaxation methods for delay differential equations

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In the talk we present delay dependent error estimates for the iterative processes

$$x'_{k+1}(t) = F(t, x_{k+1}(t), x_k(t), x_k(\cdot)), \quad k = 0, 1, \ldots, \quad t \in J = [0, T],$$
$$x_{k+1}(t) = g(t), \quad t \in [-h, 0]$$

for solving the initial value problems for systems of equations of the form

$$\begin{aligned} x'(t) &= f(t, x(t), x(\cdot)), \qquad t \in J = [0, T], \\ x(t) &= g(t), \quad t \in [-h, 0], \quad h > 0, \end{aligned}$$

where $f(t, x, y(\cdot)) = F(t, x, x, y(\cdot))$. For F we assume the one-sided Lipschitz condition with respect to the second argument

$$(F(t, x, y, z) - F(t, \bar{x}, y, z), x - \bar{x}) \leq m(t)\|x - \bar{x}\|^2$$

and the Lipschitz conditions with respect to the last two arguments

$$\|F(t, x, y, z) - F(t, x, \bar{y}, \bar{z})\| \leq K(t)\|y - \bar{y}\| + L(t)\|z - \bar{z}\|_{\beta(t)}$$

with some continuous nondecreasing function β such that $0 \leq \beta(t) \leq t$, where $\|y\|_t = \max_{-h \leq s \leq t} \|y(s)\|$ for $t \in J$. When x_* is the exact solution of our initial value problem and $u_k(t) = \max_{0 \leq s \leq t} \|x_*(s) - x_k(s)\|$ then we obtain the following results.
If $m(t) < 0$, $K(t) \equiv 0$, $P_*(t) = \max_{0 \leq s \leq t} \left[\frac{L(s)}{-m(s)}\right]$, then

$$u_k(t) \leq u_0(\beta^k(t)) \left(\prod_{i=0}^{k-1} P_*\left(\beta^i(t)\right)\right) H_k(t),$$

where the sequence $\langle H_k \rangle$ of functions depends on m and satisfies the conditions $0 \leq H_k(t) \leq 1$, $\lim_{k \to +\infty} H_k(t) = 0$.

Another result reads: if $m(t) \geq 0$ and $K(t) \equiv 0$, then

$$u_k(t) \leq \left(\prod_{i=0}^{k-1} L(\beta^i(t))\right) \exp\left(\int_0^t m(s)ds\right) u_0(\beta^k(t)) \int_0^t \Phi_{k-1}(1)(s)ds, \quad t \in J,$$

for $k = 1, 2, \dots$, with a suitably defined sequence $\langle \Phi_k(1) \rangle$ depending on β.
We have also the following result: if $\overline{m}(t) = \max(0, m(t))$, then

$$u_k(t) \leq \frac{t^k}{k!} \exp\left(\int_0^t \overline{m}(\tau)d\tau\right) \left(\sum_{i=0}^{k} \left[\binom{k}{i} K^{k-i}(t) L_*^i(t) u_0(\beta^i(t))\right]\right), \quad k = 0, 1, \dots ,$$

where $L_*(t) = \sup_{0 \leq s \leq t} \left[L(s) \exp\left(-\int_{\beta(s)}^s \overline{m}(\tau)d\tau\right)\right]$.

On equations with delay depending on solution

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Consider the following equation

$$\dot{x}(t) + \sum_{i=1}^{m} A_i(t) x(t - (H_i x)(t)) = f(t), \quad t \in [0, \infty), \tag{1}$$

$$x(\xi) = \varphi(\xi), \quad \xi < 0.$$

Assume that the elements of the $n \times n$ matrices $A_i, \quad i = 1, 2, ..., m$, are measurable and essentially bounded on $[0, \infty)$, the function $\varphi : (-\infty, 0] \to \mathbf{R^n}$ is continuous, the function $f : [0, \infty) \to \mathbf{R^n}$ is measurable and essentially bounded. Suppose also that the operations $H_i, \quad i = 1, 2, ..., m$, put into correspondence to each absolutely continuous function $z : [0, \infty) \to \mathbf{R^n}$ scalar measurable essentially bounded functions $H_i z : [0, \infty) \to \mathbf{R}, \quad i = 1, 2, ..., m$. An absolutely continuous function $x : [0, \infty) \to \mathbf{R^n}$, is called a solution of equation (1) if it satisfies the equation almost everywhere.

Along with (1) let us consider a linear equation

$$(\mathcal{L}_x y)(t) \equiv \dot{y}(t) + \sum_{i=1}^{m} A_i(t) y(t - (H_i x)(t)) = \psi(t), \quad t \in [0, \infty), \tag{2}$$

$$y(\xi) = 0, \quad \xi < 0.$$

Here $x : [0, \infty) \to \mathbf{R^n}$ is an absolutely continuous function, $\psi : [0, \infty) \to \mathbf{R^n}$ is measurable and essentially bounded.

As a solution of (2) we understand an absolutely continuous function $y : [0, \infty) \to \mathbf{R^n}$, which satisfies (2) almost everywhere.

We suggest an approach for investigation of the nonlinear equation (1). Its' essence is stressed out by the following result.

Let a function $x : [0, \infty) \to \mathbf{R^n}$ *be a solution of (1), then it is also a solution of (2), where*

$$\psi(t) = f(t) - \sum_{i=1}^{m} A_i(t)\tilde{\varphi}(t - (H_i x)(t)), \tag{3}$$

$$\tilde{\varphi} = \begin{cases} \varphi(t), & t < 0, \\ 0, & t \geq 0. \end{cases}$$

Collocation methods for computation of periodic solutions of retarded functional differential equations

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A periodic solution of a differential equation can be computed as the solution of a periodic two-point boundary value problem (BVP). The periodic BVP is a finite-dimensional problem for ordinary differential equations (ODEs) (a point has to be found) and it is an infinite-dimensional problem for functional differential equations (FDEs) (a function segment has to be found). The most successful method for the numerical solution of periodic BVPs for ODEs is the well known collocation method using piecewise polynomials. As far as we know, no prior numerical work on the application of the collocation technique (not based on approximation by truncated Fourier series) for solving periodic BVPs for FDEs exists. Furthermore, the corresponding theoretical results (convergence, asymptotic error of numerical schemes) are non existing. We show that the collocation method with piecewise polynomials is quite efficient for solving periodic BVPs for RFDEs with constant delays. Our numerous numerical experiments show numerical order of convergence of the collocation solution to the exact solution and the asymptotic error of the different collocation schemes we applied. In particular, we show that special interpolators for the approximation of the computed solution in the past recover superconvergence at mesh points which is well known for the ODEs case. Adaptive mesh selection, based on the algorithm used in the AUTO software package for ODEs, allows to compute periodic solutions with complicated profile efficiently.

On nonlinear parabolic functional differential equations

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We shall consider initial-boundary value problems for the equation of the form

$$D_t u + \sum_{|\alpha|\le m} (-1)^{|\alpha|} D_x^\alpha [f_\alpha(t,x,...,D_x^\beta u,...)] +$$
$$\sum_{|\alpha|\le m-1} D_x^\alpha [g_\alpha(t,x,...,D_x^\gamma u,...)] + \sum_{|\alpha|\le m-1} D_x^\alpha [\tilde{g}_\alpha(t,x,u)] +$$
$$\sum_{|\alpha|\le m-1} D_x^\alpha [h_\alpha(t,x,H_1(u),...,H_r(u))] = F, \quad (t,x) \in Q_{T_0} = (0,T_0)\times\Omega$$

where $\Omega \subset R^n$ is a bounded domain with sufficiently smooth boundary, $|\beta| \le m, |\gamma| \le m-1$, H_j are linear continuous operators (defined in some function spaces), functions $f_\alpha, g_\alpha, h_\alpha$ have (usual) polynomial growth in $D_x^\beta u, D_x^\gamma u, H_j(u)$ respectively; g_α, h_α are not continuous in $D_x^\gamma u, H_j(u)$ respectively (but they are locally bounded), functions $\tilde{g}_\alpha$ may be rapidly increasing in u.
We shall formulate an existence theorem on weak solutions. Further, boundedness of $\| u(t) \|_{L^2(\Omega)}$ for $t \in (0,\infty)$ will be shown. Finally, we shall formulate a result on the stabilization of solutions as $t \to \infty$.
An important problem where the conditions of our theorems are satisfied is a climate model, considered by J.I. Diaz, G. Hetzer, J. Hernández, L. Tello. The stabilization result is a joint work with J.I. Diaz and is based on arguments applied by the above mentioned authors.

Oscillation of Linear Systems

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We investigate the notion of oscillation for linear systems with discrete and continuous time. A real valued function oscillates by the definition if it has arbitrarily large zeros. A vector valued function oscillates if its composition with every linear functional oscillates.

Let T be a strongly continuous semigroup and let A denote its generator. We say that a point x oscillates if the function $T(\cdot)x$ oscillates.

We show that if the time parameter is discrete then every point oscillates if the spectrum of A has no real nonnegative values. We apply this result to oscillation of solutions of infinite difference equations.

The situation when the time is continuous is more complicated. If T is a strongly continuous group then if the spectrum of A has no nonnegative real values then every point oscillates. In the case when T is a semigroup even under this assumption not all points have to oscillate, however we show that in this case the oscillation is generic. Moreover, applying the results from [1] we obtain that if the semigroup is generated by the functional differential equation then every solution oscillates iff the generator has no real eigenvalues.

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Posters

Iterative methods for the Darboux problem for partial functional differential equations

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We consider the Darboux problem for the following hyperbolic partial functional differential equation

$$D_{xy}z(x,y) = f\left(x, y, z_{(x,y)}\right), \qquad (x,y) \in E := [0,a] \times [0,b] \tag{1}$$

$$z(x,y) = \phi(x,y), \qquad (x,y) \in E^0 := [-a_0, a] \times [-b_0, b] \setminus (0,a] \times (0,b], \tag{2}$$

where $f : E \times C^1(B, X) \to X$, $\phi : E^0 \to X$ and $B := [-a_0, 0] \times [-b_0, 0]$. The function $z_{(x,y)} : B \to X$ represents the functional dependence in (1) and it is defined by $z_{(x,y)}(t,s) = z(x+t, y+s)$, $(t,s) \in B$. If $X = \mathbb{R}$ then by the linearization of (1) we define two monotone sequences uniformly convergent to the solution of problem (1), (2). The convergence that we get is of the Newton type, which means that the difference between the m-th term of either of the approximating sequences and the solution can be estimated by $A/2^{2^m}$, where A is some constant not dependent on m.

If X is a Banach space we also prove a theorem on the convergence of the Newton method for problem (1), (2). Also in this case we get the convergence of the Newton type.

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Modal stabilization in systems with delay

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Characteristic polynomial of the linear stationary control systems without delay

$$\dot{x}(t) = Ax(t) + bu(t), \quad u(t) = c^T x(t) \tag{1}$$

has n roots. The problem of the spectrum control (modal control) is equivalent to the coefficients of the characteristic polynomial control. For systems with delay

$$\dot{x}(t) = Ax(t) + A_1 x(t-\tau) + bu(t), \quad u(t) = \sum_{j=0}^{m} c_j^T x(t - j\tau), \quad \tau > 0 \tag{2}$$

characteristic quasipolynomial

$$f_n(\lambda, e^{-\lambda\tau}) = \sum_{i=0}^{n}\sum_{j=0}^{i} p_{ij} e^{-j\lambda\tau}\lambda^{n-i}, \quad p_{00} = 1 \tag{3}$$

has countable number of roots. There is no good interconnetion between coefficients and spectrum here. The problem of modal control is set as the coefficients control problem, although the investigation of transient quality depending on coefficients cause difficulties. System stabilization up to power $\gamma > 0$ often is more advisable now. That is calculation of $c_j^T, \quad j = \overline{0,m}$, when quasipolynomial (3) roots hold $\lambda_i < -\gamma$, $i = 1, 2, \ldots$.
Theorem Let (3) be a given quasipolynomial. Then the quasipolynomial

$$\tilde{f}_n(\lambda, e^{-\lambda\tau}) = \sum_{i=0}^{n}\sum_{j=0}^{i} \tilde{p}_{ij} e^{-j\lambda\tau}\lambda^{n-i}, \quad p_{00} = 1 \tag{4}$$

$$\tilde{p}_{ij} = \sum_{s=j}^{i} p_{sj} C_{n-s}^{i-s}(N+1+\gamma)^{i-s} e^{-j(N+1+\gamma)\tau}, \quad N = \max_{i=\overline{1,n}}\{\sum_{j=0}^{i}|p_{ij}|\} \tag{5}$$

is stable up to power $\gamma > 0$.

Explicit expressions for coefficients c_j^T, $j = \overline{0,m}$ of equation $u(t)$ is obtained with formulas (4) and (5).

Equilibrium stability of price formation dynamics

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A price formation mathematical model is considered [1]. It is represented by nonlinear system of differential equations with quadratic and linear fractional right side.

$$\dot{x}_i(t) = -\frac{v_i x_i(t)}{x_i(t) + a_i} - \sum_{j=1, j\neq i}^{n} c_{ij}(x_i(t) - x_j(t)) + \frac{d_i x_i(t)}{x_i(t) - b_i} +$$

$$+ r_i x_i(t)[q_i^0 - \sum_{j=1, j\neq i}^{n} \alpha_{ij}(x_i(t) - x_j(t))], \quad i = \overline{1,n}.$$

The system can be given in vector-matrix form

$$\dot{x}(t) = \{Q - V[A + X(t)]^{-1} + D[X(t) - B]^{-1} + X(t)G\}x(t).$$

Here A, B, D, V, $X(t)$ are diagonal matrices. Conditions of solution $x(t) \equiv 0$ (balanced price) stability and lower bound of stability domain are obtained.

Theorem. Let matrix $\tilde{A} = Q - VA^{-1} - DB^{-1}$ is asymptotically stable, that is $Re\lambda_i(\tilde{A}) < 0$, $i = \overline{1,n}$. Then solution $x(t) = 0$ of the system is asymptotically stable and stability domain contains the sphere with radius

$$R = \frac{\Psi}{\sqrt{\Phi(H)}} \min\{[\Psi(H)|A^{-1} + V(A^{-1})^2|]^{-1}, [\Psi(H)|B^{-1}| + |D(B^{-1})^2|]^{-1}, |G|\},$$

$$\Psi(H) = \frac{\lambda_{\min}(C)}{6\lambda_{\max}(H)}, \quad \Phi(H) = \frac{\lambda_{\max}(H)}{\lambda_{\min}(H)}.$$

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Some Properties of Solutions of Nonlinear Difference Equations

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In the paper asymptotic properties of the solutions of nonlinear difference equation are investigated. We deal with the existence of nonoscillatory bounded and unbounded solutions.

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Oscillation of Iterative Functional Equations

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We consider behavior of solutions of iterative functional equations. Sufficient conditions for the oscillation are given.

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Some oscillation criteria for differential equations generated by delay arguments

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In this paper we investigate linear differential equations with delay arguments of the form:

$$(-1)^{\delta} L_n x(t) = \sum_{i=1}^{m} q_i(t)\, x(q_i(t)), \qquad (E_\delta)$$

where $n \geq 2, \delta \in \{1, 2, n, n+1\}$ and $L_0 x = x$, $L_i x = \frac{1}{p_i}\frac{d}{dt} L_{i-1} x, p_n \equiv 1$.

The functions p_i, g_k, $q_k : \Re_+ \longrightarrow \Re_+ = (0, \infty)$ are continuous with $g_k(t) \leq t$ on $\Re_+$, $lim_{t\to\infty} g_k(t) = \infty$ and $\int^{\infty} p_i(t) = \infty, i = 1, ..., n, k = 1, ..., m$.

The main purpose of this work is to show how the delay arguments g_i influence the oscillatory character of solutions of the above equations. We consider the equations (E_δ) depending on whether n is an even or odd number as well as the value of δ.

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2.2 Geometric Dynamics

Organizer : Hiroshi Matano

Contributed talks

The Einstein Flow, the Sigma-Constant, and Geometrization of Three-Manifolds

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For the problem of the Hamiltonian reduction of Einstein's equations on a $(3+1)$-vacuum spacetime that admits a foliation by constant mean curvature compact spacelike hypersurfaces M that satisfy certain topological restrictions, we introduce a *dimensionless non-local time-dependent* reduced Hamiltonian system $H_{\text{reduced}} : \boldsymbol{R}^{-} \times P_{\text{reduced}} \to \boldsymbol{R}$, where $\boldsymbol{R}^{-} = (-\infty, 0)$, P_{reduced} is the *reduced symplectic manifold*, and $\boldsymbol{R}^{-} \times P_{\text{reduced}}$ is the associated *reduced contact manifold*. The reduced contact variables $(\tau, \gamma, p^{TT}) \in \boldsymbol{R}^{-} \times P_{\text{reduced}}$ are given by the constant mean curvature τ of an expanding spacelike hypersurface, a Riemannian metric γ with constant scalar curvature -1, and a transverse-traceless symmetric 2-contravariant density p^{TT}.

Taking as a temporal coordinate condition $t = \frac{2}{3\tau^2}$, the reduced Hamiltonian is given by $H_{\text{reduced}}(\tau, \gamma, p^{TT}) = -\tau^3 \mathrm{vol}(M, g)$. For compact 3-manifolds of Yamabe type -1, we establish the following properties: (1) $H_{\text{reduced}}(\tau, \gamma, p^{TT})$ is a strictly monotonically decreasing function of t unless $p^{TT} = 0$ and $\gamma = \tilde{\gamma}$ is hyperbolic, at which point $H_{\text{reduced}}(\tau, \tilde{\gamma}, 0)$ is constant in time; (2) for $\tau \in \boldsymbol{R}^{-}$ fixed, $H_{\text{reduced}}(\tau, \gamma, p^{TT})$ has a critical point at $(\tilde{\gamma}, 0)$ which is unique up to isometry and which is a strict local minimum (in the non-isometric directions) of H_{reduced}; and (3) for $\tau \in \boldsymbol{R}^{-}$ fixed, the σ-constant of M is related to the reduced Hamiltonian by

$$\sigma(M) = -\tfrac{2}{3}\left(\inf_{(\gamma, p^{TT}) \in P_{\text{reduced}}} H_{\text{reduced}}(\tau, \gamma, p^{TT}) \right)^{2/3} .$$

If M is a compact hyperbolic manifold, then we conjecture that $(\tilde{\gamma}, 0)$ is a global minimum of H_{reduced}. More generally, for any compact M we conjecture that the Einstein flow generically seeks to attain the σ-constant asymptotically insofar as the reduced Hamiltonian is monotonically seeking to decay to its infimum. However, possible obstructions, such as the formation of black holes, may prevent particular solutions from asymptotically approaching $\sigma(M)$. Further applications are discussed.

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Nonlinear curvature driven evolution of plane curves

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We study the evolution of plane curves satisfying the geometric equation $v = \beta(k, \nu)$, where v is normal velocity, k and ν are curvature and the angle of the tangential vector of the plane curve Γ. As a typical example one can consider the function $\beta(k, \nu) = \gamma(\nu)|k|^{m-1}k$ where $\gamma(\nu)$ is a given anisotropy function and $m > 0$. A key tool in our study is to analyze a degenerate parabolic equation for the curvature k

$$\partial_t k = \partial_s^2 \beta(k) + \alpha \partial_s k + k^2 \beta \tag{1}$$

where α is a functional of the curvature and other geometric quantities. The geometric meaning of α is the tangential velocity of the material points belonging to Γ. We propose a nonlocal functional α keeping the ratio of the local length to the total length of the curve Γ constant with respect to time. This choice of a nontrivial tangential velocity α leads to a powerful numerical scheme.

In the case where the function β is regular we establish short time existence of a classical solution $k \in C(I, C^{2+\sigma}(S^1)) \cup C^1(I, C^\sigma(S^1))$ provided that the initial curve is smooth enough. If β is degenerate we are concerned with the fast $(0 < m < 1)$ as well as slow $(1 < m \leq 2)$ diffusion case. In the case of a slow diffusion the initial curve is supposed to have at most $2 + \frac{1}{m-1}$ order contact with its tangents at inflection points. The idea of the proof is to make use of the theory of fully nonlinear parabolic equations developed by Angenent [1] and the Nash-Moser iterative technique for obtaining a-priori bounds for the gradient of $\beta(k)$. It is similar, in technique, to the one used in the paper by Angenent, Sapiro and Tannenbaum [2]. We also present some of numerical simulations that have been obtained by using a nontrivial tangential velocity α.

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2.3 Hyperbolic Conservation Laws

Organizers : Tai-Ping Liu, Gerald Warnecke

Key note lecture

Recent Progresses in Shock Wave Theory

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There have been intensive activities in the study of shock waves in the past decade. Important progresses are made in the study of the effects of dissipations, relaxations and sources, the well-posedness problem for hyperbolic conservation laws, the regularizing properties of nonlinearity and other numerical and analytical issues. We plan to survey these developements, with emphasis on the physical phenomena and the new analytical ideas when applied to physical systems. We will also discuss open problems and speculate on possible future research directions.

Invited lectures

Convergence Rates for Relaxation Methods Approximating Conservation Laws

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This talk discusses the global error estimates for a relaxation scheme approximating scalar conservation laws. In such schemes there are two parameters: mesh size Δx and the relaxation rate ϵ. We prove that the $L^\infty(0,T;L^1(\mathbb{R}))$-norm of the error goes to zero as $\epsilon^{1/2}+(\Delta x)^{1/2}$ when both the discretization parameter ϵ and the discretization parameter Δx go to zero. We show how to do this without resorting to any restriction on the initial error. Our result shows that the initial error does not prevent the convergence of the conserved variable to the entropy solution of conservation laws, and the initial layer persists in "kinetic" variables only for a short time of order ϵ.

Nonclassical shocks for conservation laws

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Diffusive-dispersive approximations to conservation laws give rise to travelling waves that converge to shocks not verifying the classical Lax entropy condition (or Oleinik-Liu condition) and are called nonclassical shocks.

Various physical models, for which the diffusive-dispersive approximation has a physical justification, have a natural entropy. The set of all discontinuities satisfying the condition related to this single entropy are too many to have uniqueness and a selection criterion must be introduced. A kinetic relation, linked to the approximation, is used to find a unique Riemann Solver and possibly a nucleation criteria is introduced. This last appears to be natural in case of phase transition models.

Front-tracking is then used to construct solutions to the Cauchy problem. The main difficulty is the control of the total variation that may increase also in the scalar case. An equivalent functional is introduced that decreases in time along approximate solutions.

After a first result for the cubic scalar case in [1], a method for the general scalar case is described in [2]. Some assumptions are made on the kinetic function while no nucleation criteria is needed.

It is also shown that the Riemann Solvers with good properties coincides with one associated to a kinetic function of the type considered.

Finally the problem of uniqueness is investigated in [3]. It is proved that, as for the classical case, there exists under mild conditions at most one Lipschitz semigroup compatible with the Riemann solver, moreover any admissible solution satisfying a mild regularity is a semigroup trajectory.

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Stability of Viscous and Inviscid Shock Waves

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We are concerned with shock waves $(u^l, u^r; s)$ of multidimensional systems of n conservation laws. Here $u^{l,r}$ denote the left and right constant states, and s is the shock speed. The direction of propagation is *a priori* chosen along the first coordinate axis. We may consider several kinds of stability, under small initial perturbations. With increasing complexity :

- One-D stability, that is stability against perturbations in the sole x_1 direction. As is well-known, it amounts to assume $\Delta \neq 0$, where Δ is a determinant of the $n-m$ outgoing modes with m vectors representing the degrees of freedom of the shock. Here, $m = 1$ for a Lax shock, but $m > 1$ for an undercompressive shock (see [1]). All overcompressive shocks are unstable at this level.

- Multi-D stability. The theory is due to A. Majda [5] for Lax shocks and to Freistühler for undercompressive ones. At the linearized level, the shock may be strongly or weakly stable, or strongly unstable. The latter prevents from local well-posedness. The analysis requires the knowledge of zeros of an explicit analytic function : the *Lopatinski determinant*. Examples show that the sign of Δ may be involve in the nature of the shock : in full gas dynamics, $\Delta < 0$ is equivalent to strong instability.

- Viscous stability, that is asymptotic stability of a viscous profile of the shock, as a travelling wave of an approximate system. The relevance of this approach was recognized by T.-P. Liu [4]. It was shown in [3] that the sign of Δ plays a rôle in the stability, even in one space dimension. This was extended in [7] to multi-D contexts, where we show that the long-wave limit of the Evans function of the profile is nothing but the Lopatinski determinant. This shows that Majda's instability of the shock itself, which is of Hadarmard's type, implies the asymptotic instability of every possible profile.

On one hand, we shall present the latter aspect. On the other hand, we shall explain why the change of sign of Δ, as the shock varies along a given curve, is related to both weak stability and strong instability (see [6]).

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Compressible Navier-Stokes Equations with Density-Dependent Viscosity and Vacuum

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In this talk, we will present some results on the one-dimensional motion of viscous gas connecting to vacuum state when the viscosity is constant or depends on the density. When the viscosity coefficient is constant, the regularity of the solution near vacuum and the behaviour of the interface separating the vacuum and gas are given. When the viscosity coefficient μ is proportional to ρ^θ and $0 < \theta < 1/2$, where ρ is the density, the global existence and the uniqueness of weak solutions are proved.

Discrete Shock Waves and Small Divisors

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We study finite difference approximations to hyperbolic conservation laws, especially in the presence of shock waves. A small divisor problem, which is caused by discretization will be discussed.

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Contributed talks

A connection between scalar conservation laws and infinite linear systems of PDE's

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We demonstrate that solutions of most scalar conservation laws satisfy an infinite linear system of partial differential equations. This observation allows to prove an existence theorem for such infinite systems and, vice versa, to estimate the solution of the original scalar conservation law. The Hopf's equation is included in such a class of conservation laws.

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Admissibility of traveling waves in scalar balance laws

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We consider hyperbolic balance laws

$$u_t + f(u)_x = g(u), \quad f \in C^2,\ g \in C^1,$$

and their viscous approximations

$$u_t + f(u)_x = \varepsilon u_{xx} + g(u), \quad \varepsilon \ll 1.$$

Assuming that the flux is convex and that g possesses only simple zeroes, Mascia has found several types of heteroclinic traveling waves, i.e. solutions of the form $u(x,t) = u(x - st)$ that tend to constant states $u_{\pm\infty}$ as $\xi := x - st$ tends to $\pm\infty$. All these traveling waves are *entropy* traveling waves in the sense that their discontinuities satisfy an entropy condition. We discuss the question, which types of traveling waves can be obtained as limits of traveling waves of the viscous problem in the following sense: A traveling wave u_0 of the hyperbolic balance law with speed s is called *admissible* if sequences $s_n \to s$ and $\varepsilon_n \searrow 0$ exist such that the viscous problem has a traveling wave solution u_n with

$$\lim_{n\to\infty} \|u_n - u_0\|_{L^1} = 0.$$

This leads to the study of a singularly perturbed o.d.e. where the wave speed s acts as an additional parameter.
We show that not all of the entropy traveling waves are admissible.

Shocks with oscillatory tails in systems of viscous hyperbolic balance laws

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Searching for viscous shock profiles of the Riemann problem, we consider systems of hyperbolic balance laws of the form

$$u_t + f(u)_x \;=\; \epsilon^{-1} g(u) + \epsilon\,\delta\, u_{xx}$$

with $u = (u_0, u_1, \cdots, u_N) \in \mathbb{R}^{N+1}$, $\delta > 0$ fixed, and with real time t and space x. Traveling wave solutions $u = u\left(\frac{x-st}{\epsilon}\right)$ with limiting states $\lim_{\tau\to\pm\infty} u(\tau) = u^\pm$ give rise, for $\epsilon \searrow 0$, to shock solutions of the Riemann problem.

Therefore we investigate the second order traveling wave equation

$$\begin{aligned} \dot{u} &= v \\ \delta\,\dot{v} &= -g(u) + (A(u) - s)v \end{aligned}$$

and look for heteroclinic orbits between equilibria $u^\pm$ of the reaction term: $g(u^\pm) = 0$. The asymptotic behavior of viscous profiles $u(\tau)$ for $\tau \to \pm\infty$ depends on the linearization at $u = u^\pm$.

A special structure of the source term, e.g. the mixture of conservation laws with balance laws, can lead to lines or, generally, surfaces of equilibria. Investigating the change of stability along these curves of equilibria, it turns out that heteroclinic connections can have oscillating tails. This is possible even for examples with gradient flux terms where the source terms $\dot{u} = g(u)$ *alone* do not support oscillatory behavior. The interaction of flux and reaction, in contrast, is able to produce purely imaginary eigenvalues of the linearization at $u^\pm$.

Near Hopf-like bifurcation points, continua of heteroclinic orbits exist. They correspond to weak shock solutions with oscillating tails at both sides. Besides this, a fixed left state u^- can connect to an whole interval of right states u^+, with fixed traveling speed s.

The results are based on normal form theory and a spherical blow-up construction inside the center manifold near the line of equilibria.

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Stability of a Self-Similar 3-Dimensional Gas Flow

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We are concerned with inviscid gas flow in three space dimension. In the presence of symmetries, the flow becomes self-similar and the system may be reduced to ordinary differential equations. We are interested in steady flow past an infinite cone and its stability with respect to perturbation of the cone. Such a flow has a cylindrical symmetry. Let u and v represent the axial and radial components of velocity. With the additional assumption that the flow is isentropic, the compressible Euler equations are reduced to

$$(\rho u)_x + (\rho v)_y = \frac{-1}{y}(\rho v) \tag{1}$$

$$(\rho u^2 + P)_x + (\rho u v)_y = \frac{-1}{y}(\rho u v) \tag{2}$$

$$(\rho u v)_x + (\rho v^2 + P)_y = \frac{-1}{y}(\rho v^2) \tag{3}$$

$$P = P(\rho). \tag{4}$$

When a uniform supersonic flow hits the obstacle, which is an infinite cone, and the angle of opening at the vertex is sufficiently small, the conical flow can be constructed by studying self-similar solutions. The flow is deflected by an attached shock front beginning at the vertex and is continued so that the state of the air is constant on each concentric cone behind the shock cone and is parallel to the obstacle cone. Since the flow is isentropic and irrotational, the equations (1)-(4) are reduced to a system of ordinary differential equations. Such self-similar flow is suggested by Busemann [1], who gave a graphical method for obtaining them.

We consider a more realistic case when the obstacle is a perturbation of the infinite cone. The shock front and the flow behind it are conical until the expansion wave and the shock wave coming from the bendings of the obstacle interact with the conical flow and the shock front. The flow then becomes rotational and, in general, contains infinitely many interacting shock waves. The main questions are: With all these wave interactions, does a solution exist globally? Is it stable with respect to the perturbation, for finite x and also asymptotically as $x \to \infty$? We answer these questions affirmatively and show that the long-range behavior of the flow is self-similar corresponding to an infinite cone with the asymptotic angle of the perturbation. In particular, the flow between the leading shock and the obstacle tends to be irrotational and isentropic. In fact, there is a boundary layer of high concentration of vorticity and entropy variation. The width of the layer tends to zero as $x \to \infty$. This can occur, of course, only for inviscid flow; for the viscous flow the vorticity would propagate into the flow. Nevertheless, experimental evidences show that the inviscid flow still accurately represents the actual flow, Courant-Friedrichs [2]. The flow pattern eventually tends to a self-similar solution corresponding to the conical flow for the infinite cone without any deflections.

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Cauchy Problem for the Compressible Euler Equation with Damping

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We study the Cauchy problem for the system of compressible adiabatic flow through porous media in the one space dimension and the related diffusive problem, namely

$$\begin{cases} v_t - u_x = 0 \\ u_t + p(v,s)_x = -u, \\ s_t = 0, \\ (v,u,s)(x,0) = (v_0(x), u_0(x), s_0(x)), \\ (v_0,u_0,s_0)(x) \to (v_\pm, u_\pm, s_\pm), \ \ as \ x \to \pm\infty, \end{cases} \tag{1}$$

and

$$\begin{cases} \tilde{v}_t = -p(\tilde{v},s)_{xx} \\ \tilde{u} = -p(\tilde{v},s)_x \\ s_t = 0, \\ \tilde{v}(x,0) = \tilde{v}_0(x), \ \ s(x,0) = s_0(x), \\ \tilde{v}_0(\pm\infty) = v_\pm, \ \ s_0(\pm\infty) = s_\pm. \end{cases} \tag{2}$$

In this paper, we consider the following two cases

Case1: $(v_\pm, s_\pm)$ satisfy $p(v_-,s_-) = p(v_+,s_+) = \bar{p}$;

Case 2: $s_- = s_+ = \bar{s}$.

By introducing the new approach which combines the usual energy methods with special L^1-estimate and with the use of weighted norms, we can solve the problem (2) in detail. The global existence and large time behavior for the classical solutions for both two cases are proved. The asymptotic states for the solutions are given by stationary solution for case 1 and by similarity solution for case 2 respectively. Thanks to our new approach, the almost optimal convergence rates are obtained.

Then, we establish the global existence and large time behavior for the smooth solutions to the problem (1) in both cases by comparing the solutions of (1) to those of

(2) via and energy estimate methods. It is also shown that the problem (1) can be approximated very well time-asymptotically by (2) which obtained from (1) by Darcy's law. The optimal decay rates for the difference between the solutions of (1) and those of (2) are then proved by weighted energy estimates.

Our results for case 1 generalized the results in [1] for more general cases with in addition the decay rates. The results for case 2 strongly improved the results in [2] by removing the key technique condition in [2](referred in [2] as condition V) and giving the optimal decay rates.

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2.4 Hyperbolic Wave Equations

Organizers : Frank Duzaar, Joseph Grotowski

Key note lecture

The notion of hyperbolicity for Euler-Lagrange equations

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The lecture shall begin with a critical review of existing notions of hyperbolicity, i.e. Leray's notion of strict hyperbolicity and Friedrich's notion of symmetric hyperbolicity. We shall then introduce a new notion of hyperbolicity appropriate to systems of Euler-Lagrange equations. This notion is intimately connected with the notion of ellipticity in the calculus of variations (the Legendre-Hadamard condition). We shall show how the basic properties, such as the domain of dependence theorem, follow directly from the new notion, which overcomes the difficulties associated with singularities of the characteristic variety encountered by earlier approaches. Finally we shall indicate how the new approach provides a basis for the solution of problems such as the problem of global nonlinear stability of a given solution.

Invited lectures

Quasilinear Wave Equations and Strichartz Estimates

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In this paper, our aim is the proof of local wellposedness for quasilinear wave equations for initial data less regular than what is required by energy method. This implies to prove Strichartz type estimates for wave operators whose coefficients are only Lipschitz.

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Solutions of the Einstein equations with prescribed singularity structure

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The singularity theorems of Hawking and Penrose are one of the best known theoretical results in general relativity. They assert the existence of spacetime singularities under general circumstances but say nothing about the structure of these singularities. The proofs of these theorems involve differential geometry and ordinary differential equations and do not use the character of the Einstein equations as a quaslinear hyperbolic system. It seems unavoidable that the theory of hyperbolic equations must be used in order to obtain more precise information on this matter. One approach to this problem is to attempt to construct solutions of the Einstein equations with a prescribed type of singularity. Heuristic considerations indicate that the singularities could have a complicated oscillatory structure in general and this suggests difficulties in constructing them. The same heuristics indicates, however, that in the presence of a scalar field things simplify dramatically, the oscillatory behaviour being replaced by uniform blow-up.

Motivated by these considerations, Lars Andersson and the author have studied the Einstein-scalar field system from this point of view. The unknowns are a four-dimensional Lorentzian metric g and a real-valued function ϕ (the scalar field) and the equations are $\mathrm{Ric}(g) = \nabla\phi \otimes \nabla\phi$, where $\mathrm{Ric}(g)$ is the Ricci curvature of the metric g. It was shown that there exists a large class of solutions of these equations with singularities of blow-up type which can be described in considerable detail. The solution of the system of partial differential equations is described approximately near the blow-up hypersurface by a system of ordinary differential equations. (This is known in the relativity literature as the velocity dominated approximation.) The solutions constructed depend on a certain class of free data. These are required to be analytic but are otherwise general.

The analytical part of the proof is similar to that used in a more special problem in [1], the fundamental idea going back to Baouendi and Goulaouic [2].

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Contributed talks

Bounded Solutions and Blow-up in Nonlinear Wave Equations

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We consider a Klein-Gordon equation with linear dissipation and a nonlinear source term

$$u_{tt} - \Delta u + mu + \delta u_t = f(u),$$

in a bounded domain $\Omega \subset \Re^n$, with $m > -\lambda =$ first eingenvalue of $-\Delta$ in $H_0^1(\Omega), \delta > 0, f(u) = \mu u|u|^{r-2}, \mu > 0, r > 2$, and with homogeneous Dirichlet boundary condition

$$u = 0, \;\; in \;\; \partial\Omega,$$

and initial conditions

$$u(0) = u_0, \;\; u_t(0) = v_0.$$

We give necessary and sufficient conditions for globality and boundedness, and for nonglobality and blow-up of solutions. Also, we characterize those solutions which tend to the nonzero equilibria as well as to the zero equilibrium as $t \to \infty$. We improve previous results of Cazenave [1], Ikehata [2] and Goergiev and Todorova [3].

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Asymptotic behaviour of solutions to a class of semilinear hyperbolc systems.

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In this talk the long time asymptotic behavior of solutions of semilinear symmetric hyperbolic system including Maxwell's equations and the scalar wave-equation in an arbitraty spatial domain are investigated. The possibly nonlinear damping term may vanish on a certain subset of the domain. Decay in the weak topology for suitable initial-states is shown. In the case that the nonlinear damping is in addition montone, also strong local L^q-convergence is shown.

These results are applied to the initial-boundary-value-problem for Maxwell's equations in an exterior domain. It is shown that the solution behaves like a free-space-solution (in whole space with constant coefficients) provided that the initial-state is orthogonal to all stationary states. Here a L^p-regularity-theorem for Maxwell's equations and some commutator-estimates are used.

On the Attractors of Hamiltonian Nonlinear Wave Equations

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We consider the asymptotics in long-time limits, $t \to \pm\infty$, of finite energy solutions to Hamiltonian nonlinear wave equations in the whole space. For "generic" equations the solutions converge to an attractor which is the set of all stationary states. The convergence holds in the Fréchet topology defined by local energy seminorms. The convergence is established for general 1D nonlinear wave equations with nonlinear term concentrated at a finite segment [1], for 3D scalar wave equation coupled to a particle [2], and for 3D Maxwell-Lorentz system with a charge [3]. In [4] the soliton-like asymptotics is established for finite energy solutions to the translation-invariant system from [2]. In [5] the stability of the soliton manifold and the adiabatic effective dynamics are proved for the soliton-like solutions to the system from [2] with slowly varying potential. The related physical problems are discussed in [6].

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Nonlocal problem with integral conditions for hyperbolic equation

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Consider the equation

$$Lu \equiv u_{xy} + A(x,y)u_x + B(x,y)u_y + C(x,y)u = f(x,y)$$

in the rectangle $D = \{(x,y) : \ 0 < x < a, \ 0 < y < b\}$.

Nonlocal integral conditions arise when one studies certain problems of plasma physics [1], heat conduction [2], dynamics of ground waters [3]. Motivated by this, we study the following problem:

$$Lu = f, \ \int_0^\alpha u(x,y)dx = \psi(y), \ \int_0^\beta u(x,y)dy = \phi(x),$$

where $\phi(x), \psi(y)$ are given, $0 < \alpha < a, \ 0 < \beta < b$. If $\alpha = a, \ \beta = b$ the existence and uniqueness of generalized solution are proved [4].

Introducing a new unknown function $w(x,y)$, we reduce the equation $Lu = f$ to a loaded equation $\bar{L}w = F$, while the integral conditions take the form

$$\int_0^a w(x,y)dx = 0, \ \int_0^b w(x,y)dy = 0.$$

The existence and uniqueness of the solution in Sobolev space $H^1(D)$ are proved. The proof is based on a-priori estimates and a complete continuity property of the operator, generated by the problem.

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Semilinear wave equations on pseudo-Riemannian manifolds: stability and motion of non-topological solitons.

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The starting point is the equation:

$$\frac{\partial^2 \phi}{\partial t^2} - \Delta\phi + m^2\phi - \beta(|\phi|)\phi = 0 \qquad (*)$$

for a function $\phi : \mathbb{R}^{1+n} \to \mathbb{C}$. This equation supports a class of solitary wave solutions, called *non-topological solitons* ([2]), of the form

$$\phi(t, x) = e^{i\omega t} f_\omega(x), \qquad (**)$$

where f_ω is the solution of a constrained minimisation problem ([3]); in the case $\beta(f) = |f|^{p-1}$ the function f_ω can also be characterised as an "optimiser" of the Gagliardo-Nirenberg inequality ([4]). The stability properties of these solutions have been investigated in both the physics and mathematics literature (see [1],[2] and references therein); an interesting feature is the $\omega-$dependence of the stability conditions. A *modulational* approach to stability theory is developed, which gives stronger results than those obtained previously; in particular, the solution to the Cauchy problem for (*), with initial data close to a Lorentz transform of (**), remains uniformly close to the corresponding point on an explicitly determined curve in the space of Poincare transforms of (**). This stability holds for an approriate range of ω, which may, in the case $\beta(f) = |f|^{p-1}$, be determined sharply from linear theory, using information gleaned from the Gagliardo-Nirenberg characterisation of f_ω.

The main motivation for the present development of this technique is to provide a basis for an investigation of solitons in the equation

$$L_g\phi + m^2\phi - \beta(|\phi|)\phi = 0 \qquad (C)$$

where L_g is the covariant d'Alembertian determined by a pseudo-Riemannian metric g. It is proved that under the rescaling $g \to \epsilon^{-2} g$, with ϵ small, there exist solutions to (C) in which the solitons (**) are concentrated on time-like geodesics. This generalises a previous one dimensional theorem ([5]).

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On discrete models of the wave equation

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The purpose of this work is to present a generalization of Dezin's results [1] dealing with a discrete model of the Laplace operator for the case of pseudo-Euclidean space. The present work is a direct continuation of [2].

Let $\mathbb{R}^n_1$ be pseudo-Euclidean space of type $(1, n-1)$, i.e. $\mathbb{R}^n$ with the Lorentz metric. If we take $n = 4$ we obtain Minkowski space. We construct a combinatorial model of $\mathbb{R}^n_1$ as the complex $C(\mathbb{R}^n_1)$ with boundary operator ∂. Then the dual complex $K(\mathbb{R}^n_1)$ to $C(\mathbb{R}^n_1)$ can be considered as a space of discrete forms. The coboundary operator d^c on $K(\mathbb{R}^n_1)$ is a discrete analog of the exterior differential d. The definition of d^c does not depends on a metric and coincides with the one for the combinatorial model of Euclidean space [1]. But to define a discrete analog of the Hodge star operator $*$ we must take into account a metric. We define a discrete analog of the codifferential δ as the formal adjoint operator (denote δ^c) to d^c. Note that δ^c depends from a metric too. The combinatorial Laplacian is defined by

$$\Delta^c = d^c\delta^c + \delta^c d^c$$

Clearly, the generalized Laplacian with respect to the Lorentz metric on $K(\mathbb{R}^n_1)$ is a discrete analog of the wave operator. The Cauchy problem and boundary value problems for this discrete model of the wave equation are studied.

Within our intrinsic approach, we may write in terms of the generalized Laplacian a discrete model of the Klein-Gordon equation. Likewise, using the combinatorial model of Minkowski space we can describe a discrete model of the Maxwell equations. An important generalization of these equations leads immediately to a discrete model of the so-called Yang-Mills equations of the quantum field theory.

It should be possible to consider one more discrete model in which "time" is the continuous parameter. The latter fact means that a discrete model of the wave equation is constructed on the complex $K(\mathbb{R}^{n-1})$ with the Euclidean metric and $K(\mathbb{R}^{n-1})$ is depending from the continuous parameter t. In this case a system of the second order ordinary differential equations by t is a discrete analog of the wave equation.

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Global Dynamics of Nonlinear Hyperbolic Equations With Non-Monotone Damping

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Many challenging issues concerning global dynamics emerge from nonlinear hyperbolic equations with a kind of weak dissipation such as localized damping, non-monotone damping, or boundary damping. In this work, we study a nonlinear wave equation

$$u_{tt} + Au + g\left(u_t\right) + f(u) = h,$$

where typically A represents $-\nu\Delta$ over a bounded 2-D domain with homogeneous Dirichlet boundary condition, $g\left(u_t\right) = -\alpha u_t + \beta u_t^3$ with constants α and $\beta > 0$ is a non-monotonedamping, f is a Caratheodory function (or simply a polynomial) satisfying the usual power growth condition and the asymptotic sign condition, and $h \in L^\infty\left(0,\infty; L^2(\Omega)\right)$ is a given input. This equation has been used as a model in dynamic analysis and stabilizing control of the galloping vibration of power lines of electric transmission. Due to the difficulty caused by the nonlinear and non-monotone damping, the long-term behavior of solutions remains open to a large extent. The following results are obtained. First, the local existence and uniqueness of weak and strong solutions are established by the Bubnov-Galerkin approach and the regularity analysis is conducted. Second, by an asymptotical bootstrap method in dealing with a Lyapunov-like functional $E\left(u, u_t\right)$, it is proved that for any initial data in the energy space the weak solution exists globally and remains bounded based on the differential inequality

$$\frac{d}{dt}E\left(u,u_t\right) \leq -\frac{\epsilon}{2}E\left(u,u_t\right) + C_0\beta\left\|u_t\right\|_{L^4}^3\left(\epsilon\|u\|_{H^1} - C_1\right) + C_2(\epsilon) + \left(1+\alpha^{-1}\right)\|h\|_\infty.$$

Moreover, it is proved that the solution semiflow is dissipative in terms of the existence of absorbing sets. Third, with some restriction that h is time-invariant and $\beta > c\alpha$ for some constant c, it is shown that thesolution semiflow $S(t)$ is uniformly κ-contracting as it satisfies

$$\|S(t)w_1 - S(t)w_2\| \leq \psi(t)\left\|w_1 - w_2\right\| + \rho_t\left(w_1, w_2\right), \quad t > \tau \geq 0,$$

where $\psi(t) \to 0$ and ρ_t is a precompact pseudometric on the energy space. Finally it is proved that there exists a global attractor for the semiflow generated by this nonlinear hyperbolic equation.

Posters

The global Cauchy problem for the critical non-linear wave equation in variable coefficients

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In this work, we study the global existence of both smooth and strong solutions of the critical wave equation in variable coefficients in three dimension of space

$$(E)\ \ \Box_A u + \mid u \mid^4 u = \partial_t^2 u - div(A(x).\nabla_x u) + \mid u \mid^4 u = 0, \ \ I\!R_t \times I\!R_x^3.$$

where A is a regular function valued in the space of 3×3 positive definite matrices which is the identity outside a compact set of $I\!R^3$.

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2.5 Hysteresis

Organizer : Jürgen Sprekels

Key note lecture

P.D.E.s with hysteresis operators

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Hysteresis appears in several phenomena, in physics, engineering, chemistry, biology, economics, and so on. A systematic investigation of its mathematical properties began only in the 1970s, see [1].

Hysteresis relations can be represented by a *black box,* which transforms an input function u into an output function w. This is formalized through the notion of *hysteresis operator* $\mathcal{F} : u \mapsto w$, acting between spaces of time dependent functions, e.g. $C^0([0,T])$. *Rate independence* appears as the basic property; this means that if $\mathcal{F} : u \mapsto w$ then $\mathcal{F} : u \circ \varphi \mapsto w \circ \varphi$, for any increasing time diffeomorphism $\varphi : [0,T] \to [0,T]$.

For instance, the model studied by Duhem a century ago corresponds to the transformation $u \mapsto w$ defined by the following Cauchy problem:

$$\begin{cases} \frac{dw}{dt} = g_1(u,w)\left(\frac{du}{dt}\right)^+ - g_2(u,w)\left(\frac{du}{dt}\right)^- & \text{in }]0,T[, \\ w(0) = w^0; \end{cases} \tag{1}$$

here g_1 and g_2 are given continuous functions and w^0 is a datum.

The classical hysteresis models due to Prandtl, Ishlinskiĭ, Preisach will also be outlined, and some of their properties (continuity, monotonicity and others) discussed.

We then briefly deal with the following P.D.E., which arises as a simplified model of (scalar) ferromagnetic hysteresis:

$$\frac{\partial}{\partial t}[u + \mathcal{F}(u)] - \frac{\partial^2 u}{\partial x^2} = f \qquad \text{in }]a,b[\times]0,T[; \tag{2}$$

here $\mathcal{F}$ is a hysteresis operator and f is a given function. Under suitable assumptions, the corresponding initial and boundary value problem is well-posed in appropriate function spaces.

Details and an extended bibliography can be found in [2].

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Invited lectures

Asymptotically stable almost-periodic oscillations in systems with hysteresis nonlinearities

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We consider a perturbed ODE system

$$x' = f(t,x) + \varepsilon g(x, z(t)),$$

where the perturbation includes the hysteretic element (Γ being a hysteresis operator)

$$z(t) = (\Gamma[t_0, z(t_0)]Lx)(t).$$

We assume that the unperturbed system ($\varepsilon = 0$) has an asymptotically stable almost-periodic solution. We present sufficient conditions for the perturbed system to have an asymptotically stable almost-periodic solution. We illustrate the result with an example from feedback control.

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Elastic Contact Problems and the Stop Operator

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Given a closed convex set $K \subset \mathbb{R}^n$, a forcing function $f : [0, \infty) \to \mathbb{R}^n$, and an initial value $u(0) \in K$, we look for a function u such that $u(t) \in K$ for all t, and $u'(t)$ is as close as possible to $f'(t)$, i.e., $f'(t) - u'(t)$ lies in the normal cone of K at the point $u(t)$. It is well known that this problem admits a unique solution. The operator $\mathcal{S}$ mapping $(u(0), f) \in K \times W^{1,p}([0, \infty), \mathbb{R}^n)$ into $u \in W^{1,p}$ is called the stop operator.

It is known that the stop operator is continuous in many function spaces, but it is not Lipschitz continuous in the spaces $W^{1,p}$ for $1 < p < \infty$. The question of Lipschitz continuity in $W^{1,1}$ has been settled only recently. The stop operator is globally Lipschitz continuous if K is a convex polyhedron. If K has $W^{2,\infty}$-smooth boundary, then the stop operator is locally Lipschitzian. There are examples of domains K with $\mathcal{C}^1$-boundaries where $\mathcal{S}$ is not locally Lipschitz continuous.

As an application we utilize the Lipschitz continuity of the stop operator on a polyhedron to prove wellposedness of a two-point initial-boundary value problem modeling the dynamics of a linearly elastic beam with unilateral contact conditions.

Homogenization of Scalar Wave Equation with Hysteresis Operator

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Longitudinal vibrations of the elastoplastic rod are modelled by scalar wave equation

$$\rho\, u_{xx} = (\mathcal{F}[u_x])_x + f\,,$$

with constitutive stress-strain relation of elastoplastic material in the form of Ishlinskii operator

$$\sigma = \mathcal{F}[e] := \eta_\infty e - \int_0^\infty \mathcal{S}_r[e]\, \mathrm{d}\eta(r)$$

where $\mathcal{S}_r$ is the stop operator. The equation is completed by convenient initial and boundary conditions.

The material of the rod is heterogeneous i. e. spatially dependent, which means spatially dependent material data

$$\rho = \rho(x) \qquad \eta = \eta(x, r)\,.$$

Properties of spatially dependent Ishlinskii operator are studied; existence, uniqueness and estimates of the solution to this initial boundary value problem are proved.

Homogenization problem consists in considering a sequence of materials with periodic structure with diminishing period, which means a sequence of problems with periodic functions ρ^ε, η^ε in the constitutive relation having diminishing period $\varepsilon \to 0$

$$\rho^\varepsilon(x) = \rho\left(\frac{x}{\varepsilon}\right), \qquad \eta^\varepsilon(x,r) = \eta\left(\frac{x}{\varepsilon}, r\right),$$

where $\rho(y)$ and $\eta(y,r)$ are functions periodic in y. The corresponding solutions u^ε converge to a function u^0 which is a solution to the so-called homogenized problem. The form of the homogenized problem is derived and convergence $u^\varepsilon \to u^0$ is proved.

Hysteresis operators in modeling of phase transitions

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Among mathematical methods designed for solving problems in phase transitions modeling, the method of hysteresis operators is one of the most recent ones. Its development is connected with a systematic effort of applied mathematicians (see [1,3,4,6]) to set up an appropriate theoretical framework for describing rate-independent hysteresis phenomena independently of their physical nature. For complex physical problems like those arising in phase transitions, the hysteresis theory makes it possible to replace analytically nonsmooth constraints (represented e. g. by rate-independent evolution variational inequalities) by more regular objects (hysteresis operators) in suitable function spaces. It turns out that, at least in some cases, this may lead to a considerable simplification in the analysis of the problem.

The aim of this contribution is to present some extensions to the results published in [5] for the phase-field type system

$$\begin{cases} \mu(\theta)\, w_t + f_1[w] + \theta\, f_2[w] = 0\,, \\ (\theta + F_1[w])_t - \Delta\theta = \psi(x,t,\theta)\,, \end{cases}$$

motivated by the Frémond-Visintin formulation (see [2]) of the relaxed Stefan problem, where the order parameter w and the absolute temperature θ are the unknowns of the problem, μ and ψ are given functions and f_1, f_2, F_1 are hysteresis operators, and to discuss further issues.

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Posters

Monte Carlo study of the mixed spin Ising ferromagnetic model with a ferrimagnetic surface

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The three-dimensional mixed spin 1/2 and spin 1 Ising model with competing surface and bulk exchange interactions is studied. Within the frame work of the Monte Carlo simulation, the phase diagrams are investigated, they exhibit qualitatively interesting features. The effects of the surface and the bulk crystal field interactions on the phase diagrams and in particular for surface ordering, are also examined. It is also shown that such system can exhibit a variety of phase transitions.

2.6 Large Domains

Organizers : Alexander Mielke, Guido Schneider

Key note lecture

The long wave limit for the water wave problem

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I will describe the derivation and justification of approximating equations for long waves moving on the surface of an inviscid, irrotational fluid in an infinitely long canal without surface tension. A variety of equations, such as the Korteweg-de Vries or Boussinesq equations, have been derived to formally approximate the evolution of such waves.

Using recently developed techniques for justifying modulation equations one can can show rigorously that to the order of approximation considered in formal derivations of the KdV or Boussinesq equations the solutions of the water wave problem split up into two wave packets, one moving left and one moving right, where each of these wave packets evolve independently as a solution of the Korteweg-de Vries equation. In particular, this means that to the order of approximation considered, such counterpropagating waves do not interact with each other. In addition to the fact that this approximation can be rigorously justified, it also has the advantage that the approximating equations are well posed and completely integrable. This work was done in collaboration with Guido Schneider.

Invited lectures

Dimension and Entropy for parabolic PDE's in Infinite Domains

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This is a report on work with P. Collet. We define the topological entropy per unit volume in parabolic PDE's such as the complex Ginzburg-Landau equation, and show that it exists, and is bounded by the upper Hausdorff dimension times the maximal expansion rate. We then give a constructive implementation of a bound on the inertial range of such equations.

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Loss of Stability in the Magnetic Benard Problem

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We study the loss of stability of the trivial equilibrium $v_0 = \vartheta_0 = h_0 = p_0 = 0$ of the system

$$\begin{aligned}
\partial_t v &= \nu\Delta v + \nabla p + SB\partial_z h + S(h\nabla)h - (v\nabla)v + g\alpha k\vartheta \ , \\
\partial_t \vartheta &= \chi\Delta\vartheta + (T_0 - T_1)v_3 - (v\nabla)\vartheta \ , \\
\partial_t h &= r\Delta h + B\partial_z v + (h\nabla)v - (v\Delta)h \ , \quad div\ v = div\ h = 0 \ .
\end{aligned}$$

Here $k = (0,0,1)^t$, $v = (v_1, v_2, v_3)$, $h = (h_1, h_2, h_3)$, while $\nu, \chi, r, \alpha, g, S$ are constants related to the physical setting. (x, y, z) ranges over the infinite plate $\Omega = R^2 \times \left(-\frac{1}{2}, \frac{1}{2}\right)$. We assume Dirichlet conditions for v, ϑ and stress-free conditions for h, i.e. $\partial_z h_1 =$

$\partial_z h_2 = h_3 = 0$ on $\partial\Omega$. The above system describes a conducting fluid in Ω, subject to a temperature gradient induced by the temperature T_0 at $z = -\frac{1}{2}$ and $T_1 < T_0$ at $z = \frac{1}{2}$. Moreover the fluid is subject to a constant magnetic field $H_0 = (0, 0, B)$ parallel to the z-axis. This induces a magnetic field h, coupled to v via the above system. It is our aim to discuss the stability of the trivial solution under perurbations which belong to $L^2(\Omega)$, in particular its dependence on B and $T_0 - T_1$. This point of view is justified by recent results, which establish among others its equivalence with a stability notion due to W. Eckhaus.

Rolls and Modulating Pulses in the Swift–Hohenberg equation

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We consider stationary spatially periodic solutions of the Swift–Hohenberg equation $u_t = -(1+\partial_x^2)^2 u + \varepsilon^2 u - u^3$, $u = u(t,x) \in \mathbb{R}, t > 0, x \in \mathbb{R}^d, 0 < \varepsilon \ll 1$. The family of latex stationary rolls is given by $\{u_{\varepsilon,k}(x) = 2\sqrt{(\varepsilon^2 - (1-k^2)^2)/3}\cos(x_1 + \phi) + \mathcal{O}(\varepsilon^2) : k^2 - 1 \in (-\varepsilon, +\varepsilon), \phi \in S^1\}$. We present results on the nonlinear diffusive stability of Eckhaus–stable stationary rolls in two dimensions ($d = 2$) with respect to perturbations in polynomially weighted spaces [5]. The method consists in combining the results from [3] on the linearization around a roll with renormalization theory in Bloch wave space [1,4].

Next we consider a modified, spatially periodic Swift–Hohenberg equation, which can be considered as a model problem for pattern formation over weakly oscillating domains [2]. In the associated modulation equation the S^1 symmetry is broken. We consider the existence and stability of modulating pulse solutions [6]. These solutions consist of a pulse–like envelope modulating a spatially and temporarely oscillating wave train. They are constructed by means of spatial dynamics and center manifold theory.

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Contributed talks

Energy flow in extended gradient systems

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We study a class of systems on unbounded domain whose restriction to a bounded domain is a gradient system. Typical example is the real Ginzburg-Landau equation

$$u_t = u_{xx} + u - u^3, \tag{1}$$

$u(x,t) : \mathbb{R} \times \mathbb{R}^+ \to \mathbb{R}$, for which all the results apply (other examples include some 1D and 2D parabolic PDE's, damped hyperbolic PDE's, as well as lattice dynamics such as gradient dynamics of Frenkel-Kontorova models).

Extended gradient systems. We propose "axioms" of an extended gradient semiflow: relations between the energy density and energy current (satisfied e.g. by (1) on $C_{b,u}(\mathbb{R})$ in sup-norm as well as localised topologies). We deduce: non-existence of periodic orbits (with period $T > 0$), and that for each x non-stationary, the average return time of any orbit to any neighbourhood of x is ∞. If the phase space is compact (e.g. localised topology in (1)), all Borel invariant probability measures are supported on the set of equilibria, and the ω-limit set of each point contains an equilibrium.

Additional spatial structure. If we assume that the semiflow commutes with a group of spatial translations, the induced semiflow on the space of translationally invariant Borel probability measures is gradient. We deduce that, given any translationally invariant measure μ on the phase space, ω-limit set of μ-almost every initial condition consists of equilibria.

Applications. We suggest applications of the results to variational problems, in particular to construction of orbits and invariant measures of Hamiltonian dynamical systems.

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2.7 Lattice Dynamical Systems

Organizer : Shui-Nee Chow

Key note lecture

Traveling Waves in Lattice Dynamical Systems

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We discuss recent results in the theory of lattice differential equations. Such equations yield continuous-time infinite-dimensional dynamical systems which possess a discrete spatial structure modeled on a lattice. The systems we consider, generally over a higher-dimensional lattice such as $\mathbb{Z}^d \subseteq \mathbb{R}^d$, are the simplest nontrivial ones which incorporate both local nonlinear dynamics and short-range interactions. Of particular interest are traveling wave solutions connecting equilibria which may either be spatially homogeneous or exhibit regular patterns. Also of interest are the effects of anisotropy of the lattice, as well as imperfections in the lattice.

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Invited lectures

Pinning and propagation for waves for bistable lattice systems with long range interaction

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We study an Ising-like model for phase transitions, incorporating long-range interactions. The dynamics of the state is according to an l_2 gradient flow for the Helmholtz free energy. We construct traveling and stationary waves and give criteria for propagation and pinning of a wave. Some results concerning uniqueness and stability of the wave are given.

Dimer automata and edge processes

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We study a class of lattice dynamical systems where the geometric space is a grid (lattice) or, more generally, a graph, the local state space is finite (in most obvious examples the states are "0" and "1", i.e., white and black), the action is local, stochastic, and asynchronous. The motivation for these systems is the following: Deterministic synchronous cellular automata have been proposed as ultimate discretizations of systems modeling interactions and spatial spread. But these systems cannot accomodate the exchange of the contents of two neighboring cells nor a random walk.

To get a true discrete analogue of reaction diffusion equations on lattices we have defined "dimer automata", i.e., lattice dynamical systems where the deterministic local function selects a cell according to a Poisson process and evaluates two cells at a time. These systems can be classified and their dynamical behavior can be characterized in terms of a parameter space. For these systems mean field approximations can be derived which yield a close connection to differential equations and which reproduce experimentally sampled data with astonishing accuracy.

This concept has been generalized in two ways. 1. The regular grid is replaced by an arbitrary graph (topological properties of the graph play some role). 2. The deterministic local function is replaced by a convex combination (in stochastic sense) of such functions.

The resulting class of "edge processes" forms a polyhedron in some parameter space the extremal points of which are (a subset of seven) of the deterministic "dimer systems" mentioned above. The qualitative behavior of the edge processes can be characterized in terms of the theory of stochastic processes. The theory of these systems is put in relation to interacting particle systems in general and to geometric processes studied by T.E.Harris in the seventies.

Finally some findings on two-species systems on lattices are presented.

(joint work with Birgitt Schönfisch, Christoph Bandt, Frank Kriese)

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Cellular Neural Networks: Pattern and Waves

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In this talk I would like to report some results about the patterns and waves of Cellular Neural Networks (CNN). In section 1 we study the stationary solutions of CNN by using the method of iteration map. When the map is one-dimensional, the spatial entropy of stable stationary solutions can be obtained explicitly. When the map is two dimensional, then the Smale's horseshoe is also constructed. In section 2 we consider the travelling wave solutions of lattice dynamical system and prove the existence and multiplicity of monotone travelling wave solutions for some lattice differential equations by using monotone iteration method. In section 3 we illustrate the applications of these results to CNN.

Discrete Breathers

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Discrete breathers are time-periodic spatially localised vibrations of networks of oscillators. Several developments on this topic will be reported, including:

1. Existence of discrete breathers in systems with Euclidean invariance;
2. An effective Hamiltonian for approximate dynamics of generalised multi-breathers [1];
3. Numerical evidence for an almost continuous connection from discrete breathers to Anderson modes [2].

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Contributed talks

Travelling Wave Solutions for some Quasilinear Parabolic Equations

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We study the existence and uniqueness of travelling wave solutions for some quasilinear reaction-diffusion equations and their related discretized systems. These equations arise in various applications such as Mathematical Biology.

Differential equations describing the evolution of densities in particle systems

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The main objects of consideration are nearest neighbour interacting particle systems η_t on homogeneous directed graphs $G = (V, E)$. In this framework homogenity means that for each edge there exists at least one automorphism of the graph mapping this edge to a fixed "root" edge. For each edge we fix one of these automorphisms. By ordering the set of edges we get a sequence of automorphisms.

For each configuration η of the interacting particle system we define distributions of configurations on finite subgraphs $G' \subset G$ as the spatial mean with respect to the fixed sequence of automorphisms. For the subgraph $G' = \{v\}$ consisting of one vertex $v \in V$ this distribution can be interpreted as the density of particles in the configuration η.

We show that existence of all finite distributions for a starting configuration η is sufficient for almost sure existence of all finite distributions for the configuration $\eta_t,\ t \geq 0$.

Now we describe the time evolution of the density of particles by a differential equation. Here the time change depends only on the parameters of the interacting particle system and the density of edge markings (that is the density of configurations on the next level subgraph G' consisting of one edge in E).

Some interesting consequences of this relation are presented, as for example the estimation of the parameter for the contact process.

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Complete Stability for a Class of Lattice Dynamical Systems

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We consider a class of lattice dynamical systems originated from cellular neural networks with symmetric feedback templates. In the vector field of this class the state vector and the output vector is related through a sigmoidal nonlinear function. For two types of sigmoidal functions Liapunov functions have been constructed in the literature. Complete stability is studied for these systems using LaSalle's invariant principles on the Liapunov functions. The purpose of this presentation is twofold. The first one is to construct Liapunov functions for other kinds of sigmoidal functions. For example, for a type which is a mixture of the two types described in the literature. The second one is to consider the necessity for condition of isolation of equilibrium in concluding the complete stability. We shall show that for these sigmoidal functions, including the previously studied ones, the condition that every equilibrium is isolated is not necessary.

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Periodic motions of non-autonomous Toda Lattices

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We consider an infinite lattice of particles with a periodic external force and nearest neighbor interaction between particles

$$x_i'' + cx_i' = g_{i-1}(x_i - x_{i-1}) - g_i(x_{i+1} - x_i) + h_i(t), \qquad i \in \mathbb{Z}$$

where h_i are continuous T-periodic functions and $c \geq 0$ is a viscous friction coefficient. If $\overline{h}_i$ denote the mean value of h_i, our aim is to find conditions over such mean values leading to existence and multiplicity of T-periodic solutions.

The strategy of proof has two main steps: first, a related finite system is studied by using a change of variables that enable us a reduction to a simpler subsystem in which classical tools on topological degree can be applied. Second, a simple limiting argument leads to the conclusion, by using some a priori estimates deduced in the previous step.

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2.8 Microstructure

Organizer : Stefan Müller

Key note lecture

How applied analysis can make use of separation of scales: some examples

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Any more complex model in the sciences comes with multiple length and time scales. In many applications, the relevant parameter regime is such that these scales are clearly separated. In this situation, there is hope that the underlying mathematical problem is modular: The contribution of a small scale to the next larger scale can be read off from the solution of an auxiliary problem (which in fortunate cases is explicit).

The standard approach to investigate such a modular structure is by expansion of the solution of the full problem in powers of the (small) ratio of the scales. But this asymptotic analysis may not be applicable in a straightforward way when the description changes from small scale level to large scale level (from a free boundary to density function, say). Moreover, this asymptotic analysis becomes questionable when the solution on a large scale level develops singularities, since it relies on smoothness.

Modern mathematical techniques, mainly developed in variational theory, may help to investigate the modular structure induced by a separation of scales with more rigor than traditional asymptotic analysis. I would like to present a couple of such cases, like thin film micromagnets, Ostwald ripening or the lubrication approximation.

Invited lectures

Branched microstructures: scaling and self-similarity

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We address some properties of a scalar 2D model which has been proposed to describe microstructure in martensitic phase transformations, consisting in minimizing the bulk energy

$$E[u] = \int_0^l \int_0^h u_x^2 + \epsilon |u_{yy}|$$

where $|u_y| = 1$ a.e. and $u(0, \cdot) = 0$. Kohn and Müller[1] proved the existence of the minimizers for $\epsilon > 0$, and obtained bounds on the total energy which suggested self-similarity of the minimizer. Building upon their work, we derive a local upper bound on the energy and on the minimizer itself, and show that the minimizer u is asymptotically self-similar, in the sense that the sequence

$$u^j(x,y) = \theta^{-2j/3} u(\theta^j x, \theta^{2j/3} y)$$

$(0 < \theta < 1)$ has a strongly converging subsequence in $W^{1,2}$.

References

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Hypersurfaces with mean curvature given by a trace

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We consider smooth, oriented $n-$hypersurfaces $\Sigma_j = \partial E_j$ with interior E_j whose mean curvature is given by the trace of a function in the ambient space $u_j \in W^{1,p}(\mathbb{R}^{n+1})$

$$\vec{\mathbf{H}}_{\Sigma_j} = u_j \ \nu_{E_j} \quad \text{on } \Sigma_j, \tag{1}$$

where ν_{E_j} denotes the inner normal of Σ_j . We investigate (1) when $\Sigma_j \to \Sigma$ weakly as varifolds and prove that Σ is an integral $n-$varifold with bounded first variation which still satisfies (1) for $u_j \to u,\ E_j \to E$. p has to satisfy

$$p > \frac{1}{2}(n+1)$$

and $p \geq \frac{4}{3}$ if $n = 1$. The difficulty is that in the limit several layers can meet at Σ which creates cancellations of the mean curvature.

References

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A mathematical formulation of rate independent phase transformations using an extremum principle

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A mathematical model for the study of the rate independent mesoscopic behavior of shape memory alloys is presented. The model meets two basic requirements:

1. Nontrivial hysteresis loops occur, i.e. history dependence is reflected,

2. threedimensional microstructures are taken into account.

An abstract framework for rate independent processes in the spirit of the first requirement is presented. Ingrediences are a state space $P \subset X$ where X is a Banach space, a time dependent potential $I : [0,T] \times P \to \mathbb{R}_{\geq}$ and a dissipation functional $\Delta : X \to \mathbb{R}_{\geq}$, which is convex and homogeneous of degree 1. Functions $c : [0,T] \to P$ are denoted as admissible process if for every $t \in [0,T]$, $a \in P$ the stability inequality

$$\Delta(a - c(t)) + I(t,a) - I(t,c(t)) \geq 0$$

is satisfied and the energy inequality

$$I(0,c(0)) - I(T,c(T)) - \int_0^T \{\Delta(\dot{c}) - \partial_t I(t,c(t))\}\,\mathrm{d}t \geq 0$$

holds. Qualitative properties, existence and uniqueness of admissible processes are discussed in simple cases.

In a second step energies describing materials which can undergo phase transformations are relaxed in such a way that they fit into the previously developed framework. Due to the generality of the formulation the existence of admissible processes can be expected in simple cases.

Continuum Limits of Step Flow Models

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We will present various continuum limits in the regime of partial differential equations (PDEs) that arise from the Burton-Cabrera-Frank type step flow models [1]. The motivation and goal is to provide a mathematical model for epitaxial thin film growth and to understand the bridging of various length scales in surface growth phenomena. We concentrate on 1+1 dimensions.

The hierarchy of models we consider can take into the account of the diffusion of terrace adatom, attachment and detachment, edge adatom density, bulk diffusion and multiple species. The PDEs take the form of a coupled diffusion equation for the adatom density and a Hamilton-Jacobi equation for the thin film height profile. Nucleation phenomena is introduced at the top terraces by imposing appropriate boundary conditions for the PDEs.

The equations provide a description of the thin film growth in the mesoscopic scale but they can be potentially linked to the more macroscopic motions laws.

References

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Contributed talks

Numerical approach to the inverse homogenization problem

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Homogenization theory is a tool for describing effective properties of fine mixtures of materials. It is extensively used in the study of composite materials, in topology optimization and in structural design. We deal with the problem of finding composites with extremal elastic properties, that is, given two materials, finding an optimal way to mix them, in order for the composite to satisfy certain requirements, like maximum bulk modulus or extreme Poisson ratio. This is called "inverse homogenization problem", and has been treated in several papers; see [3], [4], [5]. In order to solve this problem, attention is restricted to periodic mixtures only, and then one looks for an optimal arrangement of the two materials in the periodicity cell.

In the existing literature a "black and white squares" approach is used, that is, the cell is divided in squares, a number is associated to each square, zero for one material, one for the other material. Then those numbers are allowed to take any value in $[0, 1]$ by using an artificial interpolation law between the two materials; after applying the optimization procedure a penalty method is applied to get rid of the intermediate densities. Then one has to find a way to eliminate the "checkerboard effect".

Our approach is different; it generalizes to the elastic framework the methods described in [1] and [2]. We use classical shape optimization techniques in the periodicity cell, employing a variable mesh. We simply move the boundary between the zones filled with the two materials in such a way to optimize the elastic properties of the composite. Consequently, we have no need to use an artificial interpolation law between the two base materials, or to penalize the intermediate densities, neither we have to deal with the checkerboard effect.

Our method has the disadvantage that one has to decide from the beginning the number of inclusions to be optimized. The algorithm cannot create any new inclusion or destroy an existing one.

Several numerical examples are presented.

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Quasiconvexity Conditions and Stability of Classes of Lipschitz Mappings

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Let E_k^m be the space of symmetric k-linear maps from $\mathbb{R}^N$ into $\mathbb{R}^m$. For a Borel function $f : E_k^m \to \mathbb{R}$ let

$$f^{qc}(v) := \{\int_{[0,1]^N} f(v + D^k w(x)\, dx : w \in C^\infty(\mathbb{R}^N; \mathbb{R}^m),\, w(y+z) = w(y),\, y \in \mathbb{R}^N,\, z \in \mathbb{Z}^N\}$$

denote the *quasiconvex envelop* of f at $v \in E_k^m$ [1]. Here $D^k w(x)$ stands for the differential of order k of a mapping w at $x \in \mathbb{R}^N$. A set $K \subset E_k^m$ is called *quasiconvex* if $K = \{v \in E_k^m : \operatorname{dist}^{qc}(v, K) = 0\}$.

Let K be a compact set in E_k^m. We consider the class of all continuous $W_{loc}^{k,\infty}$-solutions $u : U \to \mathbb{R}^m$ of the relation

$$D^k u \in K \text{ a.e. in } U, \tag{1}$$

where U is a domain (open connected set) in $\mathbb{R}^N$. For a quasiconvex compact set K this class is stable in the C^{k-1}-norm in the following sense.

Theorem. *Let K be a quasiconvex compact set in E_k^m. There exists a function $\alpha = \alpha_K : [0, +\infty) \times (0, 1) \to [0, +\infty)$ such that* (i) $\lim_{\varepsilon \to 0} \alpha(\varepsilon, \rho) = \alpha(0, \rho) = 0$ *if* $0 < \rho < 1$; (ii) *if V is a domain in $\mathbb{R}^N$ and $v : V \to \mathbb{R}^m$ is a continuous mapping of the class $W_{loc}^{k,\infty}$ with $\|\operatorname{dist}(v(\cdot), K)\|_{L^\infty(V)} < \infty$, then for every bounded domain U with* $\operatorname{diam}_{inn} U < \infty$ *whose neighborhood $\{x \in \mathbb{R}^N,\ \operatorname{dist}(x, U) < \frac{1-\rho}{\rho} \operatorname{diam} U\}$ lies in V there is a continuous $W_{loc}^{k,\infty}$-solution $u : U \to \mathbb{R}^m$ of* (1) *satisfying the inequality*

$$\sum\nolimits_{j=1}^{k} (\operatorname{diam}_{inn} U)^{j-k} \|D^j (v - u)\|_{C(U)} \leq \alpha(\|\operatorname{dist}(v(\cdot), K)\|_{L^\infty(V)}, \rho) \operatorname{diam} U.$$

By $\operatorname{diam}_{inn} U$ *we denote the diameter of a domain U calculated in its intrinsic metric.*

One application of the theorem is the study of ω-stability in the C-norm for classes of Lipschitz mappings. The concept of ω-stability was proposed by A. P. Kopylov [2] as a generalization of the stability property of the class I_N of isometric mappings $u : U \subset \mathbb{R}^n \to \mathbb{R}^n$. The stability of I_N in this context was proved by F. John [3]. Note that in [3] the class I_N was studied in connection with an analysis of linearizations of elasticity problems.

The research was supported by the INTAS grant 97–10170 and by the RFBR grant 99–01–0051.

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Spectral problems in a periodic thick multi-structure with concentrated masses

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Convergence theorems and asymptotic estimates (as $\varepsilon \to 0$) are proved for eigenvalues and eigenfunctions of the main boundary-value problems for the Laplace operator in a plane thick periodic junction Ω_ε with concentrated masses on thin channels. This junction consists of the junction's body

$$\Omega_0 = \{x \in \mathbb{R}^2 : 0 < x_1 < a,\ 0 < x_2 < \gamma(x_1)\},$$

and a large number N of the thin channels

$$G_\varepsilon^j = \{x \in \mathbb{R}^2 : |x_1 - \varepsilon\,(j+1/2)| < \varepsilon h/2,\ x_2 \in (-1,0]\}, \quad j = 0,1,\ldots,N-1.$$

Here $\gamma \in C^\infty([0,a])$, $0 < \gamma_0 = \min_{x_1\in[0,a]} \gamma(x_1)$; h is a fixed number from the interval $(0,1)$; N is a large positive integer, therefore, $\varepsilon = a/N$ is a small discrete parameter which characterizes the difference between the thin channels and their thickness.

The density ρ_ε of the junction is order $O(\varepsilon^{-\alpha})$, $\alpha \geq 0$, on the thin channels (the concentrated masses if $\alpha > 0$), and $O(1)$ outside of them. Spectral problems with different boundary conditions (Dirichlet, Neumann) on the boundary Υ_ε of the thin channels are considered. So, for the spectral problem

$$\begin{array}{rcll} -\Delta_x\, u(\varepsilon,x) & = & \lambda(\varepsilon)\ \rho_\varepsilon(x) u(\varepsilon,x), & x \in \Omega_\varepsilon, \\ \partial_\nu\, u(\varepsilon,x) & = & 0, & x \in \partial\Omega_\varepsilon \cap \{x : x_2 \geq 0\}, \\ u(\varepsilon,x) & = & 0, & x \in \Upsilon_\varepsilon = \partial\Omega_\varepsilon \cap \{x : x_2 < 0\}, \end{array} \tag{1}$$

with the Dirichlet conditions on Υ_ε, there are three qualitatively different cases in the asymptotic behavior of the eigenvalues and the eigenfunctions of problem (1): $0 \leq \alpha < 2$, $\alpha = 2$, $\alpha > 2$.

In the first case, the energy of the free vibrations is concentrated in the junction's body, and the eigenvalues of problem (1) have the following asymptotics

$$\lambda_n(\varepsilon) = \mu_n + O(\varepsilon) \quad \text{as} \quad \varepsilon \to 0,$$

where μ_n is an eigenvalue of some mixed boundary-value problem for the Laplace operator in the junction's body Ω_0.

In the second case, the energy is concentrated both in the junction's body and in the thin channels. The eigenvalues tend to the corresponding eigenvalues of some discontinuous self-adjoint operator-function. Furthermore, they are split into a countable family of series, the limits of the eigenvalues from each series belong to a finite interval and these intervals are mutually disjoint.

In the third case, the energy of the free vibrations is concentrated in the thin channels; all the eigenvalues tend to zero, and have the asymptotics

$$\lambda_n(\varepsilon) = \varepsilon^{\alpha-2}\Lambda_0 + \varepsilon^\alpha \beta_n + O(\varepsilon^{\alpha+1}) \quad \text{as} \quad \varepsilon \to 0.$$

Existence and relaxation results in the class of anti-plane shear deformations

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In this talk we discuss existence and relaxation results for the problem

$$J(u) = \int_\Omega L(x,u(x),Du(x))dx \to \min, u\Big|_{\partial\Omega} = f \tag{1}$$

in the class of Sobolev functions $u : \Omega \to \mathbf{R}$.

Standard assumptions in Nonlinear Elasticity lead to the requirement

$$L \geq \alpha |Du|^p + \gamma, \alpha > 0, p > n. \tag{2}$$

It turns out that under this requirement the relaxation result holds [1].

Theorem *Assume that $L : \Omega \times \mathbf{R} \times \mathbf{R}^n \to \mathbf{R} \cup \{\infty\}$ satisfies the requirement (2) and assume that it is bounded in a neighborhood of each point $(x, u, v) \in \Omega \times \mathbf{R} \times \mathbf{R}^n$, where it takes finite value.*

Then for each function $u_0 \in W^{1,p}(\Omega)$ with $\tilde{J}(u_0) < \infty$, where $\tilde{J}$ is the integral functional with the integrand obtained by convexification of L with respect to Du, there exists a sequence u_k such that u_k converges weakly in $W^{1,p}$ to u_0, $u_k\big|_{\partial\Omega} = u_0\big|_{\partial\Omega}$, and $J(u_k) \to \tilde{J}(u_0)$.

This theorem shows that to establish existence in (1) one still has to follow the standard scheme to find a solution of the relaxed problem along which the values of the original and the relaxed integrands coincide.

In the homogeneous case $L = L(Du)$ it is easy to show that the requirement *for each $v \in \mathbf{R}^n$ either $\partial L(v) \neq \emptyset$ or there exist $v_1, \ldots, v_q \in \mathbf{R}^n$ such that v belongs to the interior of the convex hull of $\{v_1, \ldots, v_q\}$ and $\cap_{i=1}^{q} \partial L(v_i) \neq \emptyset$* is both necessary and sufficient for all boundary value problems (1) to have a solution, see [2].

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Constrained Young measures

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Several problems envolving non linear effects induced by homogenization have been studied since the importance of the subject was pointed out by L. Tartar. At the same time he introduced new appropriated methods crucial for the future development on the subject, like mixing the "old" tool of Young measures with the new tool of compensated compactness (see [4]) or like developping the new tool of H-measures (see [5]).

We inscribe our contribution in the effort to describe such nonlinear effects by answering the following simplified question proposed in [6]:

Given two sequences, u_n on $\omega_1 \times \omega_2$ and v_n on ω_2 such that the constrain $v_n = \frac{1}{|\omega_1|} \int_{\omega_1} u_n(x, y) dx$ holds, what relationship do we have between their associated Young measures? Is the condition sufficient?

Under appropriate hypotheses our main result gives necessary and sufficient conditions for the above problem. Namely, we prove that the following statements are equivalent:

1) ν and μ are probability measures on $[\alpha, \beta]$ such that $\mu \prec \nu$;

2) there exists a sequence (u_n), $u_n : \omega_1 \times \omega_2 \to [\alpha, \beta]$ which gives rise to Young measure $\nu \otimes dx \otimes dy$ and the sequence (v_n) defined by $v_n(y) := \frac{1}{|\omega_1|} \int_{\omega_1} u_n(x, y)dx$ gives rise to Young measure $\mu \otimes dy$.

The order relation $\prec$ on the set of probability measures on $[\alpha, \beta]$, is a version of that introduced by G. H. Hardy, J. E. Littlewood and G. E. Pólya in [1] and is defined by: $\mu \prec \nu$ if and only if $\int_\alpha^\beta \phi(z)d\mu(z) \leq \int_\alpha^\beta \phi(z)d\nu(z)$, for all continuous and convex functions ϕ.

The characterizations of the order relation $\prec$ in terms of doubly stochastic operators and rearrangements (J. V. Ryff [2] and [3]) are employed.

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Posters

Time Dependent Young Measure Solutions for an Elasticity Equation with Diffusion

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A system of a nonconvex elasticity equation possibly coupled with a nonconvex diffusion equation is studied. The aim of this system is to describe crystals consisting of different chemical ingredients which can produce microstructures.
In particular we consider the initial boundary value problem:

$$\begin{aligned} u_{tt}(x,t) - \operatorname{div} S(\nabla u(x,t), c(x,t)) &= 0, \\ c_t(x,t) - \operatorname{div} K(\nabla c(x,t), u(x,t)) &= 0, \end{aligned}$$

where $x \in \Omega \subset \mathbb{R}^n$ and $t \geq 0$, u denotes the displacement vector, c denotes the chemical concentrations and S and K are the nonlinear material functions.
In the variational case we can prove – assuming a growth condition and certain regularities for S and K – an existence theorem for the class of time dependent Young measure solutions.

This existence theorem extends a result of Demoulini [1] for the nonconvex wave equation. Its proof uses time discretization and relaxation methods. An inportant ingredient is the fact that the quasiconvexification of a $\mathcal{C}^1$-function is in $\mathcal{C}^1$ as recently shown by Ball, Kirchheim and Kristensen [2].
Further properties of the Young measure solutions were studied.

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2.9 Nonlinear Functional Analysis

Organizers : Hansjörg Kielhöfer, Jean L. Mawhin

Key note lecture

Applications of critical point theory to variational problems on $\mathbb{R}^n$

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In this talk we will discuss some recent advances in critical point theory and their applications to elliptic problems on $\mathbb{R}^n$.

The abstract set up deals with the existence of critical points of C^2 functionals of the type

$$f_\epsilon(u) = f_0(u) + \epsilon G(u), \qquad u \in E \tag{1}$$

where E is a Hilbert space and f_0 possesses a finite dimensional, possibly non compact manifold Z of critical points. Suppose that Z is non degenerate (in the sense that $KerD^2 f_0(z) = T_z Z$ for all $z \in Z$) and let Γ denote the functional G constrained on Z. Then near any *topologically stable* critical point of Γ there exists a critical point of f_ϵ, provided ϵ is small enough. See [1] for the case that Z is compact and [2, 3] for the general case.

The preceding abstract setting provides a unified frame for a broad variety of problems, variational in nature. A typical example is the existence of homoclinics and chaos in dynamical systems, see [2, 4]. Other applications include: (i) the existence of semiclassical states for nonlinear Schrödinger equations with potential, see [5]; and (ii) the problem of finding a conformal metric with a prescribed scalar curvature, see [6, 7].

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Invited lectures

New Existence Theorems in Nonlinear Elastostatics

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Nonlinear elasticity deals with the deformation of solid bodies under the influence of external loadings. As such it forms one of the pillars of classical continuum physics - dating back to Cauchy. In spite of its age, properly formulated elasticity theory produces formidable nonlinear problems, the resolution of which is in many cases beyond the reach of present-day mathematical analysis. In particular, there are no general existence results in nonlinear elastostatics. In this talk we discuss the mathematical difficulties, which are a direct consequence of restrictions dictated by the physical phenomenon. Next we present some recent results of the speaker yielding global branches of classical solutions based upon a novel Leray-Schauder approach. Lastly we compare/contrast our results with the famous minimum-energy results of Ball.

Pattern Formation of the Stationary Cahn-Hilliard Model

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We investigate critical points of the free energy $E_\varepsilon(u)$ of the Cahn-Hilliard model over the unit square under the constraint of a mean value m. We show that for any fixed value m in the so-called spinodal region and to any mode of $D_u E_\varepsilon(m)$ there are critical points of $E_\varepsilon(u)$ having the characteristic symmetries and monotonicities of that mode provided $\varepsilon > 0$ is small enough. As ε tends to zero these critical points have singular limits where the symmetries and monotonicities are preserved. Therefore they form characteristic patterns for each mode.

It is remarkable that all singular limits are global minimizers of $E_0(u)$ which is not obvious since minimizing properties of the critical points of $E_\varepsilon(u)$ are not known.

Our method consists of a global bifurcation analysis of critical points of the energy $E_\varepsilon(u)$ where the bifurcation parameter is the mean value m. This new analytic approach suggests a path following device to obtain the critical points (near global minimizers) numerically.

Smooth Curves of Variational Solutions

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We consider positive solutions of the homogeneous Laplace equation $-\Delta u + \lambda u = u^q$ with Neumann boundary conditions and q subcritical. There are several families of symmetric multi-peaked solutions known, whose existence was shown by minimizing an energy functional subject to several side conditions (cf. e.g. [1]; the symmetries were discussed in [2]). In this talk we derive that these solutions in fact lie on smooth curves. We have to combine symmetry arguments, variational technics, and bifurcation theory.

Coauthor: Zhi-Qiang Wang

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Some results on the forced pendulum equation

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I present two results on the periodic problem for the forced pendulum equation [2], [3]. Both are inspired by an interesting example first analyzed in [1].

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Contributed talks

Multipoint Boundary Value Problems

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Consider the following multipoint boundary value problem for a second order ordinary differential equations

$$\left\{ \begin{array}{ll} y''(x) = f(x, y(x), y'(x)) + e(x) & x \in]0,1[\\ ax(0) + bx'(0) = 0 & \\ cx(1) + dx'(1) = \sum_{i=1}^{m} \alpha_i x(\xi_i) + \sum_{j=1}^{n} \beta_j x'(\tau_j) & \end{array} \right. \tag{1}$$

where $f : [0,1] \times R^2 \to R$ is a Caratheodory function, $e \in L^1([0,1])$. Given $\xi_i, \tau_j \in (0,1), \alpha_i, \beta_j \in R$, all of the α_i ,respectively β_j, have the same sign for $i = 1, 2, ..., m$ and $j = 1, ..., n$ with $0 < \xi_1 < ... < \xi_m < 1$, $0 < \tau_1 < ... < \tau_n < 1, a, b, c, d$, are real and $ad - bc \neq 0$.

Our objective is to prove the existence of solutions of problem (1) under suitable conditions on the nonlinearity f.

Multiple solutions of a quasilinear elliptic boundary value problems

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In this talk, we give a description of the set of solutions for the following problem:

$$\left\{ \begin{array}{ll} -\left(|u'|^{p-2} u' \right)' = |u|^{\alpha-1} u + \lambda |u|^{\beta-1} u & x \in]0,1[\\ u(0) = u(1) = 0 & \end{array} \right.$$

where $p > 1, 0 < \beta < 1 < \alpha$, and λ is a real parameter.

Theoretical and numerical study of multi-phase flows through order parameter formulation

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In this work, we study a mathematical model for multiphase flows. It was proposed for example in [2] in order to study the evolution of an alloy of two incompressible fluids in a channel under shear. It deals with a coupling of the Cahn-Hilliard equations (1)-(2) and the incompressible Navier-Stokes equations (3)-(4)

$$\frac{\partial \varphi}{\partial t} + v.\nabla \varphi - \operatorname{div}(B(\varphi)\nabla \mu) = 0, \tag{1}$$

$$\mu = -\alpha \Delta \varphi + F'(\varphi), \tag{2}$$

$$\frac{\partial v}{\partial t} - 2\ \mathrm{div}(\eta(\varphi)D(v)) + (v.\nabla)v + \nabla p = \mu\nabla\varphi, \tag{3}$$
$$\mathrm{div}(v) = 0, \tag{4}$$

where $B(\varphi)$ is a mobility coefficient, $\eta(\varphi)$ the viscosity of the alloy, and μ stands for a chemical potential in which F is the bulk free energy of the alloy.

This system is provided with Neumann boundary conditions for φ and μ, and by Dirichlet and/or periodicity conditions for the velocity. The Gibbs free energy F is caracterised by its double-well structure reprensenting the two phases of the alloy [3].

In a first part, we deal with theoretical properties of this system in the case of a channel under shear [1]. Under quite general assumptions on F, we prove the existence of global weak solutions even if the mobility is allowed to degenerate. Then, if the mobility does not degenerate, we show the existence and uniqueness of strong solutions which are global in 2D and local in 3D.

Then we prove a result of asymptotic stability in both dimensions 2 or 3 for the uniform alloy of composition $\varphi = \omega$, if ω lies in a metastable region of F [3]. More precisely we establish that if the initial condition is close enough from the equilibrium $\varphi = \omega$, and if the initial velocity is small enough, then there exists a unique global strong solution which converges to the stationary solution when t goes to infinity.

In a second part, we investigate the numerical approximation by finite differences of this model in 2D. In particular we show that even if the convective term is predominant in (1), the presence of the Cahn-Hilliard term limits the effects of the numerical diffusion of classical upwind schemes.

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On problems with nonlinear functional boundary conditions

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Boundary value problems for ordinary differential equations and functional differential equations are considered. The boundary conditions are described by nonlinear functionals. Sufficient criteria for the solvability are obtained. These criteria are applicable to essentially nonlinear systems. The case of boundary conditions given by nonlinear monotone mappings is studied. The topological structure (including arcwise connectedness) of the solution set is investigated. Boundary value problems with nonlinear boundary conditions arise, in particular, in the theory of control and differential games.

This research was partially financed by Russian Foundation for Basic Research under grant 97-01-00160.

Lower and upper solutions in the theory of ϕ – Laplacian problems

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In [2], the existence of solutions of the problem

$$-(\phi(u'(t)))' = f(t, u(t), u'(t)),\ u(a) = u(b),\ u'(a) = u'(b) \qquad (1)$$

in the sector $[\alpha, \beta]$ is proved. Here α and β are respectively a lower and an upper solution of problem (1), ϕ is an increasing homeomorphism from $\mathbb{R}$ onto $\mathbb{R}$ and f is a Carathéodory function which satisfies some Nagumo type conditions.

It is well known that, even if ϕ is the identity, existence of lower and upper solutions in the reversed order, i.e. $\alpha \geq \beta$, in general does not imply the existence of solutions of problem (1). If $\phi = id$ optimal conditions on $f(t, u, u') \equiv f(t, u)$, which guarantee the existence of solutions of problem (1) in $[\beta, \alpha]$, are known [1].

In this work, we study problem (1) with $f(t, u, u') \equiv f(t, u)$ and ϕ^{-1} a locally Lipschitz function. Existence of extremal solutions in $[\beta, \alpha]$ via the monotone method is obtained. Such results are based on anti-maximum comparison principles for the operator $-(\phi(u'))' - Mu$ $(M > 0)$, which are stronger than the classical anti-maximum principle since we need to compare two solutions of the problem

$$-(\phi(u'))'(t) - M\,u(t) = \sigma(t), \quad u(a) = u(b),\ u'(a) = u'(b).$$

These comparison results are derived from those obtained in [3] for the Neumann problem. Our results are optimal in the sense that if ϕ is the identity we obtain the best possible estimate on M given in [1]. Furthermore, we indicate functions ϕ^{-1} which are not locally Lipschitzian and f so that there exist a lower solution α, an upper solution $\beta \leq \alpha$ and there exists no solution lying between β and α.

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On the ensemble of resonance of a fourth order equation.

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In this talk we will try to give an answer to a question that appears in the book of Fučík [1]. The problem is to give a description of the set A_1 of all pairs (a, b) such that the system

$$\begin{cases} u^{iv} = au^+ - bu^- \\ u(0) = u(\pi) = u''(0) = u''(\pi) = 0, \end{cases}$$

has a non-trivial solution. Here $u^+ = \max\{u, -u\}$ and $u = u^+ - u^-$. This problem appears when one looks for a priori bounds in a problem

$$\begin{cases} u^{iv} = f(u) \\ u(0) = u(\pi) = u''(0) = u''(\pi) = 0, \end{cases}$$

where f grows linearly.

In this talk we describe the set A_1 as a countable set of analytic curves. We shall also analyze their intersections and the asymptotic behavior. This situation resembles the case of second order in which analytic curves go through the eigenvalues of the operator.

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Remarks on a periodic boundary value problem for second order impulsive integro-differential equations

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Consider the following second order integro-differential equation with impulses at fixed moments

$$\begin{array}{ll} -u''(t) = f(t, u(t), [Ku](t)), & t \in J \setminus \{t_1, t_2, \dots, t_p\}, \\ \Delta u|_{t=t_k} = I_k(u'(t_k)), & k = 1, 2, \dots, p, \\ \Delta u'|_{t=t_k} = \bar{I}_k(u(t_k)), & k = 1, 2, \dots, p, \end{array} \tag{1}$$

where $J = [0, 2\pi]$, $0 = t_0 < t_1 < t_2 < \cdots < t_p < t_{p+1} = 2\pi$, $f\colon J \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, $I_k, \bar{I}_k\colon \mathbb{R} \to \mathbb{R}$, $\Delta u|_{t=t_k} = u(t_k^+) - u(t_k^-)$, $\Delta u'|_{t=t_k} = u'(t_k^+) - u'(t_k^-)$ and K is the integral operator defined by $[Ku](t) = \int_0^T a(t,s)u(s)ds, \quad a \in C[J \times J; \mathbb{R}]$.

Note that this equation has the peculiarity that the impulses are in some sense crossed, since $\Delta u|_{t=t_k}$ depends on $u'(t_k)$ and $\Delta u'|_{t=t_k}$ depends on $u(t_k)$. Equation (1) has been considered in several papers with different boundary conditions. Here we shall study the periodic problem, i.e.,

$$u(0) = u(T), \quad u'(0) = u'(T). \tag{2}$$

In [2] problem (1)-(2) is studied without impulses in u and with linear impulses in u' (i.e. $I_k = 0$, $\bar{I}_k(x) = c_k x$). In [1] it is considered with linear impulses at u and u' (i.e. $I_k = c_k x$, $\bar{I}_k(x) = \bar{c}_k x$). Both papers are presented in the more general scheme of Banach spaces.

By establishing certain existence and comparison results, and using the monotone iterative technique, we shall obtain a criteria on the existence of minimal and maximal solutions for (1)-(2).

In the scalar case we improve and simplify results in [1] and [2]. Besides, it is easy to generalize our results for integro-differential equations in Banach spaces.

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The sweeping processes without convexity

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The sweeping process introduced by Moreau in the Seventies describes the behaviour of a moving point that is constrained both to belong in each time moment t to some closed set $K(t)$, and to have a velocity "normal" to the boundary of this set. Due to mechanical applications (such as friction and resistence) this moving point may be treated as the velocity of a particle as well. The case of convex set $K(t)$ was detaily investigated by J.J.Moreau, C.Castaing, M.Valadier, M.D.P.Monteiro Marques and other researchers. Some results were also obtained if $K(t)$ is the complement of an open convex set by a suitable way depending on t.

We propose a new approach to study the sweeping processes associated with not necessarily convex sets. Passing to the mathematical formulation, we have to find a solution of the problem

$$\left\{ \begin{array}{rcl} -\dot{x}(t) & \in & \mathbf{N}_{K(t)}(x(t)) \\ x(0) & = & x_0 \in K(0) \\ x(t) & \in & K(t) \qquad \forall t \end{array} \right. \tag{1}$$

where $\mathbf{N}_K(x)$ means the Clarke normal cone to K at the point $x \in K$, and the mapping $t \mapsto K(t)$ is supposed to be Lipschitzean. Notice that in the nonconvex case statement of the problem depends essentially on the meaning of normal vectors. Moreover, as shown by an example, the problem (1) in general can have no solutions if we consider the Bouligand normal cone in the place of the Clarke one. We present two existence results for solutions of (1) in a Hilbert space. One of them requires local compactness of the graph of $K(t)$, while other involves the assumption permitting to control the lack of convexity of $K(t)$ through some variational inequality. It seems that the last property known in the literature as φ-convexity (see, e.g., [1]) is strictly related with existence and uniqueness of the projection onto the set. Also we study the variational properties which naturally generalize φ-convexity and their relations with regularity of the closed sets. Finally, we prove uniqueness and regularity theorems for solutions of (1) under assumptions that the moving closed set $K(t)$ satisfies some of these properties.

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Polynomial solutions on matrix differential equations of Ricati type

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By introducing and using the properties of polynomial part of the expansion of a root of polynomial in scalar case, up to now polynomial solutions of Riccati differential equations and some wider classes of algebraic differential equations are described - issue studied in many papers from many various authors.

Matrix equations of Riccati type with λ-matrixes as coefficients is solved in this thesis. λ-matrix with the basic properties of previously mentioned polynomial is defined, its existing is proved in a terms of eigenvalues of the oldest matrix and than the solutions of the matrix differential equation are described and also the conditions for their existence are given.

For commutative matrices the results get the same form as in the scalar case.

Boundedness of Volterra Difference Equations

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Volterra difference equations mainly arise in the modeling process of some real phenomena or by applying a numerical method to a Volterra integral equation. Actually much of their general quantitative and specially their qualitative theory remains to be developed.

We consider the following system of Volterra difference equations

$$\Delta x(n) = A(n)x(n) + \sum_{j=n_0}^{n-1} K(n,j)x(j) + F(n),\ x(n_0) = x_0, \tag{1}$$

where $\Delta x(n) = x(n+1) - x(n)$; $A(n)$, $K(n,j)$ are $m \times m$ matrices for each $n, j \in N$, and $F : N \to R^m$ is a function.

The goal of this article is to give sufficient conditions under which the solutions of the Volterra difference Equation (1) are bounded. For this purpose, we will establish that the following linear difference Equation (2) is equivalent to the Equation (1):

$$\Delta y(n) = B(n)y(n) + L(n,n_0)x_0 + H(n),\ y(n_0) = x_0, \tag{2}$$

where

$$B(n) = A(n) - L(n,n),\ H(n) = F(n) + \sum_{j=n_0}^{n-1} L(n,j+1)F(j) \tag{3}$$

and L is an $m \times m$ matrix.

Moreover, weighted norms are used to find sufficient conditions under which Equation (1) has bounded solutions.

A multiplicity result for a floating beam equation

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We consider a strongly indefinite functional whose critical points are the weak solutions of a beam equation of the type

$$\begin{aligned} u_{tt}(x,t) + u_{xxxx}(x,t) + b[u(x,t)]^+ &= c && \text{in } [0,a] \times \mathbf{R} \\ 0 = u_{xx}(0,t) = u_{xx}(a,t) = u_{xxx}(0,t) &= u_{xxx}(a,t) && \text{for all } t \in \mathbf{R} \\ u(x,t) &= u(x,t+2\pi) && \text{for all } x \in [0,a],\ t \in \mathbf{R} \\ u(x,t) &= u(x,-t) && \text{for all } x \in [0,a],\ t \in \mathbf{R} \\ u(x,t) &= u(a-x,t) && \text{for all } x \in [0,a],\ t \in \mathbf{R} \end{aligned}$$

where b and c are positive constants and $[u]^+ = \max(0,u)$. This kind of equation, modelling a floating beam, has been studied by Lazer and Mc Kenna by means of a finite dimensional reduction. They prove the existence of two nontrivial solutions when b belongs to a certain interval.
We get the existence of two nontrivial solutions for $b > -\Lambda_1^-$, and the existence of four nontrivial solutions for $b > -\Lambda_k$ (with $k \geq 2$) and b close to Λ_k. Here we denote by Λ_i^- the sequence of the negative eigenvalues of the operator $Lu = u_{tt} + u_{xxxx}$.
In the proof we describe the geometry of a suitable associated functional and use a limit category argument.
This is a joint work with Claudio Saccon.

Lower and Upper Solutions Method for Third Order Nonlinear Periodic Problem

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We study in this work an existence result for the nonlinear third order periodic boundary value problem $x''' = f(t, x, x', x'')$ using topological degree and the method of lower and upper solutions for continuous functions. With our technique we can bound the solution and its derivative.

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Quasilinear Elliptic Equations in the whole space

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We look for positive (weak) solutions of the following problems:

$$-\Delta_p u(x) + \alpha u^{p-1}(x) = \lambda h(x) u^{q-1}(x) + g(x) u^{r-1}(x), \quad x \in \mathbb{R}^N, \qquad (P_\lambda)$$

with $1 < p < \infty$, $1 < q < p < r \leq p^*$, where $p^* = \frac{Np}{N-p}$ if $1 < p < N$, $p^* = \infty$, if $p \geq N$, λ is a real parameter, $\alpha \geq 0$, h and g verify some integrability conditions. When $p = 2$ the problem corresponds to the classical *Laplacian* and even in this case the results are new. We will study three different cases:

We will emphasize the differences of behaviour with the case of the Dirichlet problem in a bounded domain.

We would like to point out that for $p = 2$ and the critical case the problem is related to some scalar curvature problems in $\mathbb{R}^N$.

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Setting of the dual problems in mathematical physics

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A solution of boundary value problem of mathematical physics is used at calculation of physical fields. These problems are a differential corollary of conservation laws of some physical magnitudes (u — basic variable). For magnitude u in domain $V \subset R^3$ the functional equation is given:

$$L(u) = f \tag{1}$$

Let the "natural" boundary conditions are given on a surface bounding the domain

$$pn = \phi \tag{2}$$

where "fluxes" p of magnitude u are introduced with the help of operator M

$$p = M(u) \tag{3}$$

Thus

$$Div\ M = L \tag{4}$$

There is a dual setting of problem (1), (2). It consists of solution of the equations

$$Div\ p = f \tag{5}$$

being a corollary of relations (1), (4). Besides the "equations of a compatibility" are introduced:

$$P(p) = 0 \tag{6}$$

being the solvability conditions of the equation (3).

Variational principles are connected, as a rule, with each of the dual problems (1), (2) and (5), (6), (2). The simultaneous application of these variational principles allows energetically estimating a solution of the problem from above and from below.

We offer new setting of the dual problem which in most cases essentially simplifies realization of applied calculations of physical fields. It consists of solution of the functional equations

$$P(p) + A\{Div\ p - f\} = 0$$

with boundary conditions

$$P_n = \phi; \quad Div\ p - f = 0$$

The new variational principle is offered. The effective methods of a numerical realization of the problem in new setting are indicated. Special examples are included. The possible generalizations are considered.

Upper and lower solutions for first order discontinuous BVPs

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Existence of Carathéodory – type solutions for the problem

$$(P) \qquad x'(t) = f(t, x(t)) \text{ for a.e. } t \in [0, T], \quad B(x(0), x) = 0,$$

where f and B are not necessarily continuous functions, is studied in the frame of upper and lower solutions.

Condition $B(x(0), x) = 0$ is defined as an extension of the periodic boundary conditions ($x(0) = x(T)$) which involves the behavior of the solution along the whole interval $[0, T]$. For instance, under the previous formulation we will be able to consider functional conditions such as

$$x(0) = \int_0^T x(s)ds \quad \text{or} \quad x(0) = \max\nolimits_{t \in [0,T]} x(t).$$

Existence of solutions for problem (P) is studied in [1], with absolutely continuous upper and lower solutions. We shall see that the regularity of these functions can be relaxed to obtain finer results in practical situations: we shall define the concepts of upper and lower solutions in the space of functions of bounded variation, covering the useful case of piecewise absolutely continuous upper and lower solutions (see [2, 3]).

In [1] the following assumptions on f are required: $f(t, x) = q(x)g(t, x)$ where $q : \mathbb{R} \to (0, \infty)$ needs not be a continuous function, and g is a Carathéodory function (in particular, continuous with respect to its second variable). We shall see how to prove an existence result for (P) assuming no continuity hypothesis over g: essentially, this is done replacing the Carathéodory conditions by the ones introduced in [4].

Finally, we consider the situation in which the upper and the lower solution are given with no ordering. To prove a proper existence and localization result in this case we have to couple the basic ideas presented in [5] with adequate new arguments (suitable to deal with upper and lower solutions of bounded variation).

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Equivalence of two definitions of index of periodic solutions of an evolution equation of second order

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We consider the evolution equation

$$\ddot{u} + c\dot{u} + \ell u = f(t, u)$$

where ℓ is a coercive self-adjoint operator with dense domain in a Hilbert space H and f is periodic in time.

If u is a periodic solution in t of such equation, we can define the index of u in two ways: either as the fixed point of the Poincaré map or in terms of a standard functional setting for periodic problems.

We prove the equivalence of both definitions. In the proof we follow the ideas of Krasnoselskii and Zabreiko.

This is joint work with R. Ortega.

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The Fucik spectrum of general Sturm-Liouville problems

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We consider the general Sturm-Liouville boundary value problem

$$\begin{gathered} -(pu')' + qu = \alpha u^+ - \beta u^-, \quad \text{in } (0, \pi), \\ a_0 u(0) + b_0 u'(0) = 0, \quad a_1 u(\pi) + b_1 u'(\pi) = 0, \end{gathered}$$

where $u^\pm = \max\{\pm u, 0\}$. The set of points $(\alpha, \beta) \in \mathbb{R}^2$ for which this problem has a nontrivial solution is called the Fucik spectrum. When $p \equiv 1$, $q \equiv 0$, and either Dirichlet or periodic boundary conditions are imposed, the Fucik spectrum is known explicitly, and consists of a countable collection of curves, with certain geometric properties. In this talk we show that similar properties hold for the general problem above, and also for a further generalization of the Fucik spectrum. We also discuss some spectral type properties of a positively homogeneous, 'half-linear' problem, and use these results to consider the solvability of a nonlinear problem with jumping nonlinearities.

Bounded solutions for nonlinear elliptic equations in unbounded domain

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We prove the existence of a bounded, radial solution in unbounded domain of the nonlinear elliptic problem

$$\begin{aligned} &\Delta u = f(\|x\|, u) \text{ for } \|x\| > 1,\ x \in \mathbb{R}^n,\ n \geq 3 \\ &u(x) = 0 \text{ for } \|x\| = 1 \end{aligned} \tag{1}$$

under some asymptotic and sign condition on f.

Under stronger assumptions it is proved that this solution must be of constant sign.

The problem is at resonance, since the function $u(x) = 1 - \|x\|^{2-n}$ is a solution for coresponding homogeneous BVP (with zero right-hand side of (1)). The theory of coincidence degree developed in [1], [2] does not apply here, because the differential operator (with the boundary condition taken into account) is not Fredholm.

To prove the existence of solution of (1) we use perturbation method together with some fixed point theorem in the space of bounded and continuous functions.

A good survey of known results on the existence of positive solutions of (1) is in [3]. However, only solutions convergent to zero at infinity are considered therein.

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Adjoint Operators to Nonlinear Operators

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If on a nonvoid set the nonnegative identical symmetric function is given then this set is called weakly metric space. An operator which maps one weakly metric space to another weakly metric space is considered. In this situation the concepts of adjoint spaces and adjoint operator are introduced. Different properties of adjoint operators are establshed. The role of adjoint operator concept for nonlinear equation solvability is explained. More explicitly the Lipshitz operators and its adjoint operators in Banach spaces are investigated.

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2.10 Variational Methods

Organizer : Mariano Giaquinta

Key note lecture

Minimal geodesics on groups of volume-preserving maps and generalized solutions of the Euler equations

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The three-dimensional motion of an incompressible inviscid fluid is classically described by the Euler equations, but can also be seen, following Arnold, as a geodesic on a group of volume-preserving maps. Local existence and uniqueness of minimal geodesics have been established by Ebin and Marsden. In the large, for a large class of data, the existence of minimal geodesics may fail, as shown by Shnirelman. For such data, we show that the limits of approximate solutions are solutions of a suitable extension of the Euler equations or, equivalently, as sharp measure-valued solutions to the Euler equations in the sense of DiPerna and Majda.

Invited lectures

Existence of equivariant harmonic maps

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One way to obtain harmonic maps between Euclidean spheres in nontrivial homotopy classes is to reduce the effective dimension of the problem by imposing certain symmetry conditions. Several situations where this reduction leads to ODE problems have been discussed (cf. [1] for an overview), beginning with Smith's paper [3] about harmonic joins. In this talk, examples are given where the reduced problem is of higher dimension.

We minimize energy in Sobolev spaces of mappings between spheres which are equivariant with respect to certain torus actions. Doing so, we get weakly harmonic minimizers. It is now a matter of regularity theory [2] to deduce conditions under which these minimizers are actually smooth harmonic maps. These conditions imply existence of harmonic representatives of the nontrivial elements of the following homotopy groups: $\pi_{n+1}(S^n) \cong Z_2$ $(n \geq 3)$, $\pi_{n+2}(S^n) \cong Z_2$ $(n \geq 5$ odd). Moreover, each of these classes contains infinitely many harmonic maps of arbitrarily high energy.

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Critical points of the area functional by gluing technics

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We prove a 'gluing theorem', which states roughly that if two (appropriate) constant mean curvature surfaces (resp. minimal surfaces) are juxtaposed, so that their tangent planes are parallel with opposite orientation, and very close to one another, then there is a new constant mean curvature surface (resp. minimal surface) near this configuration, which is a topological connected sum of the two surfaces. Even though, gluing constructions for geometric objects are by now well understood and somewhat commonplace (e.g. in the context to CMC and minimal surfaces, the pioneering work was that of N. Kapouleas and, probably more relevant to the results here, is the recent work of S.D. Yang, in which geometric connected sums of complete minimal surfaces of finite total curvature are constructed), our procedure is both easier to understand, gives more information. What is more, the technics can be easily adapted to other problems (e.g. existence (and uniqueness) of Ginzburg-Landau vortices, existence of CMC or minimal hypersurfaces, existence of singular or regular solutions of semilinear elliptic equations, ...).

A simple context for our result is when we are given two orientable compact nondegenerate CMC surfaces, Σ_1 and Σ_2, with nonempty boundaries. Let us asume that $0 \in \Sigma_1 \cap \Sigma_2$, that $T_0\Sigma_1 = T_0\Sigma_2$ are equal to the $x\,y$-plane and have opposite orientation at this point. Then we prove that there is S_ϵ a one-parameter family of smooth, immersed CMC surfaces with boundary, for $\epsilon \in (0, \epsilon_0]$, such that, S_ϵ converges in the $\mathcal{C}^\infty$ topology to $\Sigma_1 \cup \Sigma_2$ away from the origin, and the dilated surface $\epsilon^{-1}\, S_\epsilon$ converges in the $\mathcal{C}^\infty$ topology to a catenoid with vertical axis. Moreover, the boundary of S_ϵ is the union of the boundaries of the Σ_i, each possibly transformed by a small rigid motion.

We also give sufficient conditions for S_ϵ to be embedded and finally we prove that these surfaces S_ϵ are "generically" nondegenerate (This question of nondegeneracy is quite important in the moduli space theory).

The above result can be used to establish a general connected sum theorem for complete, noncompact, embedded CMC surfaces (resp. minimal surfaces). For example, using this theorem and starting from two Delaunay surfaces, we are able to produce new embedded four-ended CMC surfaces which are quite different from the previously known examples.

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Contributed talks

Periodic solutions and rigid rotation of Ginzburg-Landau vortices

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Ginzburg-Landau vortices are finite action critical points of the functional

$$\mathcal{E}(A,\Phi)=\frac{1}{2}\int_{\mathcal{D}}|dA|^2+|D_A\Phi|^2+\frac{\lambda}{4}(|\Phi|^2-1)^2dx \qquad GL$$

where $-iA$ is an S^1 connection, Φ is a section of the associated complex line bundle and $D_A\Phi = d - iA\Phi$ is the covariant derivative. The action is invariant under an infinite dimensional group of gauge transformations $\rho(\chi):(A,\Phi)\mapsto(A+\nabla\chi,\Phi e^{i\chi})$ for smooth functions χ. The domain $\mathcal{D}$ is taken to be either S^2 or a disc in the plane. Time periodic solutions of systems describing the dynamics of Ginzburg-Landau vortices are obtained by a direct variational method. The existence of such solutions was conjectured in [1]. The systems studied include the Abelian Higgs model, which is a system of semi-linear wave equations, as well as a system of non-linear Schrödinger type, introduced in [2]. The solutions describe vortices moving in rigid rotation about one another. A previous result of this type was proved in [3] by perturbative techniques.

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The loaded differential Equations and their Applications

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The equations prescribed in $\Omega\subset R^n$ are called loaded, if they contain some operations of the traces of desired solution on manifolds (of dimension which is strongly less than n) from Closure $\overline{\Omega}$. The loaded equations are examined in Works [1], [2].

Assume we are given a bounded and regular connected open Set $\Omega\subset R^N$ and positive Number T(T is a final instant of time). Let $Q=\Omega\times(0,T), \Sigma=\partial\Omega\times(0,T), \overline{\Omega}=\Omega\cup\partial\Omega$.

Problem. Find a Solution $u(x,t)$ of the Boundary Value Problem:

$$\begin{aligned}
&\frac{\partial u}{\partial t}=\Delta u+\sum\nolimits_{k=1}^{m}M_k(x,t)u+f,\ \{x,t\}\in Q,\\
&\frac{\partial u}{\partial t}+\frac{\partial u}{\partial \vec{n}}+\sum\nolimits_{k=1}^{n-1}c_k(\sigma,t)\frac{\partial u}{\partial \vec{\tau}_k}=\sum\nolimits_{k=m+1}^{m+l}M_k(\sigma,t)u+g,\ \{\sigma,t\}\in\Sigma,\\
&u(x,0)=u_0,\ x\in\Omega,\quad u(\sigma,0)=u_1,\ \sigma\in\partial\Omega,
\end{aligned}$$

where $M_k(\theta,t)u = \int_{\Gamma_k(t)} \mathcal{M}(\theta,\xi,t) D_\xi^{\alpha_k(t)} u(\xi,t)d\xi,$

$\Gamma_k(t) - (n-1)$ – dimensional Manifolds in $\overline{\Omega}$ for $\forall t \in [0,T]$,

$D_\xi^\gamma = (1-\Delta_{|\partial\Omega})^{\gamma/2}$ – fractional integral-differential Operator of γ-order,

$\Delta_{|\partial\Omega}$ – the Laplace-Beltrami Operator,

$\vec{n}, \vec{\tau}$ – the unique outer normal and tangent vectors to $\partial\Omega$.

Theorem 1. Let $f \in L_2(0,T;\Xi^{-1}(\Omega)),\ g \in L_2(0,T;H^{-1/2}(\partial\Omega)),\ u_0 \in L_2(\Omega),\ u_1 \in L_2(\partial\Omega)$. Then the **Problem** has unique Solution $u(x,t)$ which satisfies

$$u \in L_2(0,T;H^1(\Omega)) \cap L_\infty(0,T;L_2(\Omega) \cap L_2(\partial\Omega))$$

and depends continuously from f, g, u_0, u_1.

Theorem 2. Let $f \in L_2(Q),\ g \in L_2(\Sigma),\ u_0 \in H^1(\Omega)(u_1 = u_{0|\partial\Omega})$. Then the **Problem** has unique Solution which satisfies $u \in L_\infty(0,T;H^1(\Omega)) \cap H^1(0,T;L_2(\Omega) \cap L_2(\partial\Omega))$ and depends continuously from f, g, u_0.

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On the Heat Equation with Integral Mean-Values

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Eequations with integral mean-values arise in the search of approximative solutions of partial differential equations. For example, this approach was used for the calculation of the physical and heat engineering properties of the oiling in the bearings [1].

Let $Q = \{x,t|0 < x < 1, 0 < t < T\}$. We examine the following boundary value problem:

$$\begin{cases} (\pm)v'(t) = \dfrac{\partial^2 u(x,t)}{\partial x^2} + f \text{ in } Q, \\ u(0,t) = u(1,t) = 0 \text{ in } (0,T), \\ v(0) = 0, \text{ where } v(t) = \int_0^1 u(\xi,t)d\xi, \end{cases} \tag{1}$$

$$W_1 = \{u|u \in L_2(0,T;H^2(0,1) \cap H_0^1(0,1)),\ v'(t) \in L_2(0,T)\},$$

$$W_2 = \{u|u \in L_2(0,T;H_0^1(0,1)),\ v'(t) \in L_2(0,T)\}.$$

We examine problem (1) both with the sign plus and the sign minus.

Theorem 1. For every $f \in L_2(Q)$, there is a unique $u \in W_1$ such that (1) holds.

Theorem 2. For every $f \in L_2(0,T;H^{-1}(0,1))$, there is a unique $u \in W_2$ such that (1) holds. We examine the following generalization of problem (1):

$$\frac{\partial v(x,t)}{\partial t} = \Delta u(x,t) + f(x,t) \text{ at } \Omega \times (0,T),$$

$$u(x,t) = 0 \text{ at } \partial\Omega \times (0,T), \quad v(x,0) = \varphi(x) \text{ at } \Omega,$$
$$v(x,t) \equiv \sum_{j=1}^{m} \alpha_j(x) u(x_j,t).$$

Here $\Omega \subset R^n-$ bounded Region, $\Delta-$ Laplace's Operator.

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Integral inequalities associated with Inequalities of Opial type

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We derive some new integral inequalities of the form $(*)$:

$$\int_I u|h|^p|h'|dt \le (p+1)^{p/(p+1)} \left(\int_I r|h'|^{p+1}dt\right)^{1/(p+1)} \left(\int_I s|h|^p|h'|dt\right)^{p/(p+1)}, \ h \in \ H,$$

where $I = (\alpha,\beta), -\infty \le \alpha < \beta \le \infty$, $p > 0$, H is a wide, determined class of functions h, absolutely continuous on I, satisfying the limit conditions $h(\alpha) = 0$ or $h(\beta) = 0$, the functions r, s and u are any set of functions related by the appropriate weight functions. The inequalities we examine are analogues of integral inequalities of Weyl type which are connected with integral inequalities of Hardy type [1]. To obtain the inequalities $(*)$ we use at first the uniform method of obtaining integral inequalities of Opial type of the form $(**)$:

$$\int_I s|h|^p|h'|dt \le \int_I r|h'|^{p+1}dt, \qquad h \in, H,$$

where r is an arbitrary weight function, s depends on a parameter and is chosen appropriately for r in such a way that the inequality $(**)$ holds in a wide class of functions absolutely continuous on I [2]. Next by the optimisation with respect to parameter we obtain the inequality $(*)$.

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Multiple existence of solutions for semilinear elliptic problems

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In this talk, we consider the multiple existence of semilinear elliptic problems of the form

$$\begin{aligned} -\Delta u + u &= g(x,u), \quad x \in \Omega \\ u &= 0 \qquad\quad \text{on } \partial\Omega \end{aligned}$$

where Ω is an unbounded domain in $\mathbb{R}^N$ and $g : \Omega \times \mathbb{R} \to \mathbb{R}$ is a continuous function with a superlinear growth. The existence of solutions of the problem above has been studied by many authors. In case that $g(x,t) = \mid t \mid^{p-1} t$ with $1 < p < (N+2)/(N-2)$ and $\Omega = \mathbb{R}^N$, it is well known that the problem has a unique positive solution. It is also known in this case that the problem has infinity many sign changing solutions. In this talk, we consider the existence of positive solutions and sign changing solutions for the case that g depends on x and the domain Ω is not necessarily the whole space.

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Recent results about existence of solutions to dynamic contact problems with Coulomb friction

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Despite its practical application, the theory of dynamic contact problems remains underdeveloped. The Signorini contact condition to the hyperbolic system is up–to–now solvable only for half–spaces or for one space dimension. This led to introduction of some (possibly small) physically based viscosity into the elastic problems. Moreover, the Coulomb friction law creates the problem to be nonsmooth, the friction term is nonmonotone and seems to be of the same order as the volume forces.

We shall present our recent results about the existence of solutions for the approximate Signorini condition formulated in velocities. They are based on the penalization of the contact condition and on the proof of a partial regularity of solutions to such auxiliary problem (which allows the limit procedure to the original problem). The proof, employing a certain localization technique, is based on exact trace and inverse trace theorems which allow the bounds for the admissible magnitude of the coefficient of friction to be as high as possible. The bounds depend exclusively on the viscous behaviour of the material and are remarkably better for homogeneous isotropic materials. The coefficient of friction can be solution–dependent.

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Bifurcation of homoclinic solutions for Hamiltonian systems

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We consider the following first order Hamiltonian system

$$Ju'(x) + Mu(x) - \nabla_s F(x, u(x)) = \lambda u(x)$$

where J is a $2N \times 2N$ real matrix such that

$$J = -J^T = -J^{-1} \,,$$

M is a $2N \times 2N$ real symmetric matrix such that

$$\sigma(JM) \cap i\mathbb{R} = \emptyset$$

and $F : \mathbb{R} \times \mathbb{R}^{2N} \to \mathbb{R}$ is such that

$$\lim_{|s|\to 0} \frac{F(x,s)}{|s|^2} = 0 \,.$$

We are interested in existence and bifurcation results of homoclinic solutions for a subset of such systems. By homoclinic solution, we mean a solution $u(x)$ such that $\lim_{x\to\pm\infty} u(x) = 0$.

With global and local hypotheses on F, we prove the existence of non trivial homoclinic solutions when the parameter λ is in a small neighbourhood.

Under certain local conditions on F, we find a bifurcation point for the system.

The proofs are based on variational arguments developed in the reference below.

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The shape of functions yielding best constants in Sobolev-type inequalities

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If for $p, q \in [1, \infty]$ one looks for minimizers u of

$$J_1(v) := \frac{||\nabla v||_{L^p(\Omega)}}{||v||_{L^q(\Omega)}} \qquad \text{on} \quad W^{1,p}(\Omega) \ \cap \int_\Omega v \ dx = 0 \tag{1}$$

or

$$J_2(v) := \frac{||v||_{W^{1,p}(\Omega)}}{||v||_{L^q(\Omega)}} \qquad \text{on} \quad W^{1,p}(\Omega), \tag{2}$$

then already in one space dimension, for $\Omega = [-1, 1]$ the shape of minimizers varies considerably in q, for fixed p.

As shown in [1], the minimizer u_1 of (1) changes of course sign and is odd and monotone for $q \leq 2p + 1$ and not odd but still monotone for $q > 4p - 1$. This improves results in [2] and [3].

The minimizer u_2 of (2) is always of one sign. In fact it is constant for $q \leq p$, but nonconstant, positive, strictly monotone and has exactly one inflection point for $p < q < \infty$, which moves to the boundary as $q \to \infty$. The case $q = \infty$ was treated in [4].

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Gradient flow for Willmore surfaces

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Here we report on joint work with Reiner Schätzle (Freiburg). An immersion $f : \Sigma \to \mathbb{R}^3$ of a closed, oriented surface Σ is called a Willmore surface if it is critical for the Willmore integral

$$W(f) = \int_\Sigma H^2 \, d\mu, \tag{1}$$

where $H = \frac{\kappa_1 + \kappa_2}{2}$ is the mean curvature and $d\mu$ is the induced area element. L. Simon used the direct method to prove the existence of a minimizing torus, see also [2]. Here we propose the study of the corresponding L^2 gradient flow

$$\partial_t f = (\Delta H + 2H(H^2 - K))\nu, \tag{2}$$

where ν is the unit normal and $K = \kappa_1 \kappa_2$ is the Gauß curvature. It is not clear whether (2) develops singularities in finite time and whether it converges as $t \to \infty$.
We prove a lower bound for the maximal lifespan $T > 0$ of the solution to (2) depending on how much the curvature is concentrated in L^2 for the initial surface. More precisely for $0 < \varepsilon_0 < 8\pi$ we introduce the quantity

$$r_0(f) = \inf\{r > 0 : \exists x \in \mathbb{R}^3 \text{ with } \int_{f^{-1}(B(x,r))} |A|^2 \, d\mu \geq \varepsilon_0\} \tag{3}$$

and prove the following result:
Theorem Let $f_0 : \Sigma \to \mathbb{R}^3$ satisfy $\int_\Sigma |A_0|^2 \, d\mu_0 \leq \Lambda < \infty$. There are constants $\varepsilon_0 > 0$, $c_0 < \infty$ depending only on Λ such that $T \geq c_0 \, r_{\varepsilon_0}(f_0)^4 =: T_0$. On $[0, T_0]$ the solution is estimated in terms of the initial data.
If the flow blows up at $T < \infty$, the result implies $r_{\varepsilon_0}(f) \leq \left(\frac{T-t}{c_0}\right)^{1/4} \to 0$ and thus the curvature concentrates.

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Regularity for minimizers of some variational integrals

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We consider integral functionals of the calculus of variations

$$\mathcal{F}(u,\Omega) = \int_\Omega f(Du)\ dx, \tag{1}$$

where Ω is an open subset of $\mathbb{R}^n$, $n \geq 2$, $u : \Omega \to \mathbb{R}^N$, $N \geq 1$ and the integrand f satisfies the following growth assumption

$$|\ z\ |^p \leq f(z) \leq L(1+ |\ z\ |^q), \tag{2}$$

$q \geq p \geq 2$, $z \in \mathbb{R}^{nN}$. A wide literature has been devoted to the study of regularity properties of local minimizers of (1) when $q = p$. More recently, the case $q > p$ has been considered. In particular, in the scalar case ($N = 1$) regularity of local minimizers has been proved, provided q and p are not too far apart. Counterexamples show that restrictions on the exponents p, q have to be assumed in order to have regular minimizers, [1, 2, 3]. Now we consider the vectorial case $N \geq 1$ and we deal with higher integrability. In our setting a local minimizer of $\mathcal{F}$ is a function $u \in W^{1,1}_{loc}(\Omega)$ such that

$$f(Du) \in L^1_{loc}(\Omega) \tag{3}$$

and

$$\int_{supp\varphi} f(Du)\ dx \leq \int_{supp\varphi} f(Du + D\varphi)\ dx, \tag{4}$$

for every $\varphi \in W^{1,1}(\Omega)$ with $supp\varphi \subset\subset \Omega$. By (2) and (3) we only have that $u \in W^{1,p}_{loc}(\Omega)$: we prove (see [4]) that the minimality of u boosts its integrability from $W^{1,p}_{loc}(\Omega)$ to $W^{1,q}_{loc}(\Omega)$ provided

$$q < p + 2\min\{1, \frac{p}{n}\}. \tag{5}$$

Once we filled the gap between p and q, standard technique allows us to get higher differentiability and further higher integrability.

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Closed Surfaces Minimizing the Bending Energy under Prescribed Area and Volume

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Let $\Sigma \subset \mathbb{R}^3$ be a closed surface, and h be its mean curvature whose sign is positive for convex surfaces. We consider a variational problem for the bending energy

$$W(\Sigma) = \int_\Sigma (h - c_0)^2 dS$$

under the prescribed area $A(\Sigma)$ and volume $V(\Sigma)$. Minimizers of this problem are interpreted as models of the shape of red blood cells [1,2]. Here, the constant c_0 denotes the spontaneous curvature for the red blood cell membrane. If Σ is a critical point, then it satisfies

$$\delta W(\Sigma) + \lambda_1 \delta A(\Sigma) + \lambda_2 \delta V(\Sigma) = 0,$$

where δ means the first variation, and λ_i's are the Lagrange multipliers. This is a second order elliptic partial differential equation for h.

Many results on the existence of critical points as well as their stability have been obtained numerically or by formal computations. For instance, Jenkins [3] obtained numerically bifurcating families of critical points near the sphere which are surfaces of revolution (in the case $c_0 = 0$). Subsequently Peterson [5] and Ou-Yang and Helfrich [4] carried out formal analysis of the stability/instability of the lowest-mode solution.

The purpose of this work is (i) to point out some subtleties overlooked in those formal analyses, (ii) to give a solid mathematical basis for the bifurcation analysis, and (iii) to prove, among other things, that there do exist local minimizers near the sphere unless c_0 coincides with an exceptional value.

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Multiple solutions for an elliptic equation with non–homogeneous boundary conditions

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Let us consider the nonlinear elliptic problem

$$\begin{cases} \Delta u \ + \ |u|^{p-2}\, u = 0 & \text{in } \Omega \\ u = g & \text{on } \partial\Omega \end{cases} \tag{1}$$

where Ω is an open smooth bounded subset of IR^N, $N \geq 2$, $g \in C(\partial\Omega) \cap H^{1/2}(\partial\Omega)$ and $p > 2$ is fixed.

If $g \equiv 0$, it is well known that (1) has infinitely many distinct solutions for $2 < p < \frac{2N}{N-2}$ if $N \geq 3$ or $p > 2$ if $N = 2$. In this case the energy functional is even in a Banach space, then such results have been proved by using variational methods and the properties of the genus for symmetric sets.

On the contrary, if $g \not\equiv 0$ the problem (1) loses its symmetry and the energy functional is not invariant under any group of symmetries.

However, we prove that it is possible to apply a perturbative method introduced by Rabinowitz and developed by Bahri – Berestycki and Struwe. Hence, for $p > 2$ but not too larger, the existence of infinitely many solutions of (1) with higher and higher energy can be stated (see [2]).

Further generalizations of this result are contained in [1] and [3].

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Convexity estimates for mean curvature flow of mean convex surfaces

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We consider a family $\mathcal{M}_t$ $(t \geq 0)$ of n–dimensional immersed surfaces in IR^{n+1} evolving by mean curvature flow under the assumption that the initial surface $\mathcal{M}_0$ is smooth, closed and mean convex (i.e. with nonnegative mean curvature). Then the flow is defined up to some critical time T at which the curvature becomes unbounded. Our aim is to describe the singular behaviour as $t \to T$ using rescaling techniques.

The paper [2] considered the case of surfaces satisfying a bound of the type $H \leq C/\sqrt{T-t}$ for the mean curvature H. It was shown that any limit of rescalings of such surfaces is a self-similar homothetically shrinking solution of the flow.

Here we develop a different approach, based on a–priori estimates, which can be applied without restrictions on the growth of H. We denote by $\lambda_1, \ldots, \lambda_n$ the principal curvatures of the evolving surfaces $\mathcal{M}_t$ and let S_k $(k = 1, \ldots, n)$ be the elementary symmetric polynomial of degree k evaluated in $\lambda_1, \ldots, \lambda_n$. In particular, $S_1 = H$. Then our main result is (see [3,4])

Theorem 1 *Let $\mathcal{M}_t$ be a family of surfaces evolving by mean curvature flow with $\mathcal{M}_0$ closed and mean convex. Then, for any $\eta > 0$ there exists C_η such that the estimate*

$$S_k \geq -\eta H^k - C_\eta$$

holds on $\mathcal{M}_t$ for any $t \in [0, T[$ and for all $k = 2, \ldots, n$.

The arbitrariness of η breaks the scaling invariance in the above inequality and implies that near a singularity, where H becomes unbounded, each S_k becomes nonnegative after rescaling:

Corollary 2 *Let $\mathcal{M}_t$ be a solution of mean curvature flow as in Theorem 1. Then any smooth rescaling of the singularity for $t \to T$ is convex.*

Once we know that the rescaling is convex, we can use the results of [1] about convex eternal solution of the flow to describe the profile of the surfaces which do not satisfy the growth restriction mentioned above. We show that in this case the rescaling procedure yields a translating solution of the flow.

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Some general quadratic integral inequalities of the second order

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Integral inequalities of the second order of the form

$$(1)\ \int_I sh^2 dt \leq \int_I rh''^2 dt,\ h \in H,$$

where $I = (\alpha, \beta), -\infty \leq \alpha < \beta \leq \infty$, r and s are real functions of the variable t, H is a class of functions h absolutely continuous on I, have been considered by many authors. They have been also derived by Florkiewicz and Wojteczek (see [1]) using the uniform method of obtaining integral inequalities involving a function and its derivatives.

Now we shall derive integral inequalities of the form

$$(2)\ \int_I (rh''^2 + sh'^2 + uh^2)dt \geq 0,\ h \in \hat{H},$$

using a generalization of this uniform method.

This generalization of the uniform method lets us for a given function r and auxiliary functions φ_1, φ_2 to determine directly the weight functions s and u and use them to build a wide class $\hat{H}$ of functions h for which the inequality (2) holds.

Then, using the inequality (2), we shall obtain some integral inequalities with constant weight functions, e.g. classical Hardy, Littlewood and Polya inequality of the form

$$(3)\ \int_I (h''^2 - h'^2 + h^2)dt \geq 0,$$

(see [2]) and examine the classes of functions h for which these inequallities hold.

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Bifurcation Results for a Class of Quasilinear Elliptic Systems on R^n

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In this paper we prove the existence of a branch of solutions for the system

$$\begin{aligned} -\Delta_p u &= \lambda a(x)|u|^{p-2}u + F_1(x,\lambda,u,v),\ x \in R^n \\ -\Delta_q v &= \lambda d(x)|v|^{q-2}v + F_2(x,\lambda,u,v),\ x \in R^n \end{aligned}$$

under certain conditions for p,q,F_1, F_2. This fact is achieved by applying degree theory. It was possible to carry out these methods by working between certain equivalent weighted and homogeneous Sobolev spaces.

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Posters

Direct gradient method for semilinear elliptic systems

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The subject of this paper is the numerical solution of semilinear elliptic systems

$$\begin{cases} T_i(u_1,\dots,u_r) \equiv -\mathrm{div}\,(a_i(x)\nabla u_i) + f_i(x,u_1,\dots,u_r) \;=\; g_i(x) \quad \text{in } \Omega \\ \qquad\qquad\qquad\quad Qu_i \equiv \alpha(x)u_i + \beta(x)\partial_\nu u_{i|\partial\Omega} = 0 \end{cases} \tag{1}$$

$(i = 1,\dots,r)$, where $a_i(x) \geq m > 0$ and the eigenvalues λ of the matrices $\left\{\frac{\partial f_j(x,\xi)}{\partial \xi_i}\right\}$ fulfil $0 \leq \kappa \leq \lambda \leq \kappa' + \gamma \sum_{j=1}^{r} |\xi_j|^{p-2}$. We investigate the application of the Hilbert space version of the gradient method (GM) to system (1). Whereas the usual application of the GM is done after discretization , our approach is, in contrast, to execute the iteration directly in the corresponding Sobolev space, which demands the solution of linear auxiliary problems.

It is proved that for any $u^0 = (u_1^0,\dots,u_r^0) \in H^2(\Omega)^r$ with $Qu_i = 0$ $(i = 1,\dots,r)$ in trace sense we can suitably define $M > 0$ such that the sequence

$$u_i^{k+1} = u_i^k - \frac{2}{M+m}(-\Delta)^{-1}(T_i(u^k) - g_i) \quad (k \in \mathbf{N};\ i = 1,\dots,r)$$

converges to the weak solution $u^* = (u_1^*,\dots,u_r^*)$ of (1) according to the estimate

$$\|u^k - u^*\|_{H_0^1(\Omega)^r} \;\leq\; \text{const.}\cdot \left(\frac{M-m}{M+m}\right)^k .$$

The solution of the auxiliary equations can be achieved by discretization, thus linear algebraic systems have to be solved instead of nonlinear ones obtained from discretizing (1). (A suitable FEM or FDM might be used here, similarly to the case of the Dirichlet problem.)

We examine some special domains in the case of polynomial nonlinearity when the auxiliary equations can be sol ved directly (without discretization). This includes domains that can be suitably transformed to a ball, cube or an annulus. Numerical examples of the solution of elliptic RDE equations are given.

Bifurcation of some semilinear elliptic equations

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We investigate the exact number of positive solutions of the semilinear boundary value problem

$$\left.\begin{aligned} \Delta u + f(u) \;=\; 0 \\ u_{|\partial B_R} = 0 \end{aligned}\right\} \tag{BVP}$$

where $B_R \subset \mathbf{R}^n$ is the ball centered at the origin with radius R and f is a C^2 convex function on $[0,\infty)$. Our aim is to determine the bifurcation diagram of positive solutions versus the radius R and to classify every strictly convex C^2 function according to the shape of the bifurcation diagram.

As is well-known, every positive solution of (BVP) is radially symmetric, hence it satisfies an ODE boundary value problem on the interval $[0, R]$. Then positive solutions of (BVP) can be obtained by the shooting method. We investigate the number of positive solutions via the time-map. Our aim is the describe the shape of the time-map, which is achieved through the following three characteristic properties:

- the domain of the time-map
- the limit of the time-map at the boundary points of its domain
- the monotonicity of the time-map on the maximal subintervals of its domain.

Using this the exact number of positive solutions of (BVP) can be determined.

We present different results on the above properties for $n \geq 1$ and establish the bifurcation diagram for certain convex functions. In the one-dimensional case a full characterization is given for strictly convex C^2 functions.

2.11 Viscosity Solutions

Organizer : Neil Trudinger

Key note lecture

Homogenization of Hamilton–Jacobi PDE and Hamiltonian dynamics

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I will discuss some recent work (done jointly with D. Gomes) concerning the homogenization of Hamilton–Jacobi PDE in the spirit of Lions–Papanicolaou–Varadhan. The principal new ideas concern the interpretations and implications of the “effective” Hamiltonian for the long time asymptotics of certain solutions of the related Hamiltonian ODE.

Invited lectures

Singularities of Concave Solutions to Hamilton-Jacobi Equations

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After a general introduction to the properties of the singular set (points of non-differentiability) of viscosity solutions to nonlinear first order partial differential equations, the authors investigate the structure of such a set. In particular, a new technique introduced in [1] for studying propagation of singularities will be presented and compared with earlier methods.

Finally, the special case of a Hamilton-Jacobi equation of the form

$$u_t + H(Du) = 0 \quad \text{a.e. in} \quad \mathbb{R}_+ \times \mathbb{R}^n$$

with initial condition

$$u(x,0) = u_0(x)$$

will be considered. It is well known that the solution of the above problem is concave with respect to x whenever the initial datum u_0 is concave and the Hamiltonian H is convex (see e.g. [2]). Actually, using Hopf's formula, one can prove that u is concave in $\mathbb{R}_+ \times \mathbb{R}^n$. For such a solution the authors show that singularities propagate, from any initial singular point (t_0, x_0), along a Lipschitz arc $(t, x(t))$ for all times $t \geq t_0$ up to $+\infty$. Moreover, the singular arc $x(t)$ satisfies the generalized equation of characteristics

$$\dot{x}(t) \in \nabla H(D^+u(t, x(t))) \qquad t \geq t_0 \text{ a.e.}$$

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On the rate of convergence in the homogenization of HJ equations

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The homogenization of partial differential equations can be regarded as an asymptotic process in which one looks at the limiting behaviour of solutions of a PDE with a rapidly oscillating structure as the frequency of oscillations tends to infinity.

We shall consider here fully nonlinear first order scalar partial differential equations of Hamilton-Jacobi type such as

$$u^\epsilon(x) + H(x, \frac{x}{\epsilon}, Du^\epsilon(x)) = 0 \quad \text{in } \mathbb{R}^N \tag{1}$$

where ϵ is a small positive parameter and the Hamiltonian H satisfies

$$\xi \mapsto H(x, \xi, p) \quad \text{is} \quad Z^N - \text{periodic} \tag{2}$$

for each $(x,p) \in \mathbb{R}^{2N}$. Equations of type (1) arise for example, via dynamic programming, in optimization problems for non linear control systems with rapidly oscillating dynamics. Although the importance of homogenization of Hamilton-Jacobi equations the first step on this subject was taken in 1985 by P. L. Lions, G. Papanicolau and S.R.S.Varadhan who established that, as ϵ tends to 0, the limit problem of equations (1) is given by

$$u(x) + \overline{H}(x, Du(x)) = 0 \quad , \quad x \text{ in } \mathbb{R}^N \tag{3}$$

where the effective Hamiltonian $\overline{H}$ is obtained by solving the so-called cell problem (which, in the control theoretic interpretation, can be seen as an ergodic control problem) . The next major contributions are due to L.C.Evans (1989) who developed the perturbed test functions methods for studying the homogenization problem in the framework of the theory of viscosity solutions. Further reaserch in this direction has been performed by M.C. Concordel, K. Horie and H. Ishii, O. Alvarez.

The question of estimating, in terms of ϵ , the rate of the uniform convergence of solutions of equations (1) to the solution of equation (3) has not been tackled up to now, at least in our knowledge. The purpose of this talk is to present some recent results in this direction obtained in collaboration with H. Ishii. Our main result is as follows:

Theorem Assume that H satisfies assumption (2),

$$\nu|p| - M \leq \ H(x,\xi,p) \leq \nu|p| + M \quad (x,\xi,p \in \mathbb{R}^N),$$

and $\|D\ H\|_\infty \leq M$, for some constant $M > 0$ and let u^ϵ,$u \in BUC(\mathbb{R}^N)$ be respectively the viscosity solutions of equations (1) and (3).
Then there is a constant $C > 0$ independent of $\epsilon \in (0,1)$ such that

$$|u^\epsilon(x) - u(x)| \leq C\epsilon^{1/3} \quad (x \in \mathbb{R}^N).$$

Almost Periodic Homogenization of Hamilton-Jacobi Equations

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I shall discuss the almost periodic homogenization of the Hamilton-Jacobi equation

$$\lambda u^\varepsilon(x) + H(x, x/\varepsilon, u^\varepsilon(x), Du^\varepsilon(x)) = 0 \quad \text{ in } \mathbb{R}^N.$$

Here $\lambda > 0$ is a given constant, H is a given continuous real function on $\mathbb{R}^N \times \mathbb{R}^N \times \mathbb{R} \times \mathbb{R}^N$, which is often called the Hamiltonian, u^ε represents the unknown function, and $\varepsilon > 0$ is a small parameter to be sent to zero. The question addressed here is the characterization of the limit function of u^ε as $\varepsilon \to 0$. Main assumptions on $H(x,y,r,p)$ are its coercitiveness in the p variable and its almost periodicity in the y variable. The results I shall explain describe the limit partial differential equation which the limit function should solve and show the uniform convergence of u^ε to the solution of the limit equation under the assumptions above together with some other mild assumptions.

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Hessian Measures

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Hessian measures are defined through the action of certain second order, partial differential operators on associated classes of subharmonic functions. Special cases are the distributional Laplacian acting on classical subharmonic functions and the Monge-Ampere measure of Aleksandrov, as determined by the action of the Monge-Ampere operator on the class of convex functions. In the complex case, a measure arises through the action of the complex Monge-Ampere operator on certain plurisubharmonic functions.For these and more general operators,the associated classes of subharmonic functions are the viscosity subsolutions, and one is led to the general question of how the operator acts on its viscosity subsolutions. In this talk,we present a recent result,with Xu-jia Wang,on the weak continuity of operators given as elementary symmetric functions of the eigenvalues of the Hessian matrix of second derivatives.We also explore the extension of our techniques to other classes of operators,including quasilinear operators.

Contributed talks

Propagation of Maxima and Strong Maximum Principle for Degenerate Elliptic equations

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We consider $2nd$ order partial differential equations of the form

$$F(x, u, Du, D^2u) = 0, \quad \text{in } \Omega, \tag{1}$$

where $\Omega \subseteq \mathbb{R}^N$ is an open set, $F\colon \overline{\Omega} \times \mathbb{R} \times \mathbb{R}^N \times S(N) \to \mathbb{R}$, Du corresponds to the gradient of u, D^2u corresponds to the matrix of second derivatives of u and $S(N)$ is the set of symmetric $N \times N$ matrices. We require that F satisfies a very weak ellipticity condition.

We extend a Boundary Hopf Lemma and the Strong Maximum Principle to semicontinuous viscosity subsolutions of (1). We test our assumptions on several examples involving for instance the p-Laplacian and the minimal surfaces operator. We can cover equations that are singular at $p = 0$ and very degenerate operators such as the ∞-Laplacian and some first order Hamilton-Jacobi equations. Moreover we characterize the set of propagation of maxima of the subsolutions of $2nd$ order Hamilton-Jacobi-Bellman equations. More precisely we consider $2nd$ order operators which can be represented as the supremum or the infimum of a parametrized family of linear operators. We show that the set where a maximum propagates contains the reachable set of a suitable deterministic controlled system as well the reachable set of a controlled stochastic differential equation. We also derive a Strong Maximum Principle for fully nonlinear equations satisfying a suitable version of Hörmander hypoellipticity condition.

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On the Liouville property for viscosity solutions of fully nonlinear elliptic problems

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The classical Liouville theorem states that every bounded either from above or from below harmonic function in $\mathbb{R}^N$ is constant. This property still holds for nonnegative, superharmonic functions in $\mathbb{R}^N$, provided that $N \leq 2$.

In recent years, the Liouville property has been generalized to semilinear elliptic equations stating the nonexistence of positive solutions in unbounded domains and it has been the object of a keen interest in the literature, also for its connection with the problem of the a priori bounds and the existence of positive solutions of superlinear boundary value problems in bounded domains, see e.g. [1,2,4.5] and the references therein.

I will present some results, obtained in a joint work with Fabiana Leoni (see [3]), concerning the Liouville property for nonnegative viscosity supersolutions of a class of fully nonlinear uniformly elliptic equations in $\mathbb{R}^N$. The first one regards the problem:

$$u \geq 0\,, \quad -\mathcal{M}^+_{\lambda,\Lambda}(D^2u) \geq 0 \qquad \text{in } \mathbb{R}^N\,,$$

where $\mathcal{M}^+_{\lambda,\Lambda}$, for fixed $0 < \lambda \leq \Lambda$, is the Pucci extremal operator

$$\mathcal{M}^+_{\lambda,\Lambda}(D^2u) = \sup_{A \in \mathcal{A}_{\lambda,\Lambda}} \operatorname{tr}(AD^2u)$$

with $\mathcal{A}_{\lambda,\Lambda} = \{A \in \mathcal{S}_N \ : \ \lambda|\xi|^2 \leq A\xi\cdot\xi \leq \Lambda|\xi|^2,\ \forall\,\xi \in \mathbb{R}^N\}$. I will show that u is constant provided that $N \leq 1 + \frac{\Lambda}{\lambda}$.

The second one is relative to the problem:

$$u \geq 0\,, \quad F(x, D^2u) - h(x)u^p \geq 0 \qquad \text{in } \mathbb{R}^N,$$

where F is a uniformly elliptic operator with ellipticity constants $0 < \lambda \leq \Lambda$. Under some optimal restrictions on $h \geq 0$ and p, the only solution is $u \equiv 0$.

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Lyapunov functions for infinite-dimensional systems

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We study Lyapunov functions for infinite dimensional dynamical systems governed by general maximal monotone operators. We obtain a characterization of Lyapunov pairs by means of the associated Hamilton-Jacobi partial differential equations, whose solutions are meant in viscosity sense, as evolved in works of Tataru [1] and Crandall-Lions. Our approach leads to a new sufficient condition for Lyapunov pairs, generalizing a classical result of Pazy [2].

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Gradient estimates, interior gradient blow up and comparison principle of nonlinear pde's

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A comparison principle for semicontinuous viscosity sub- and supersolutions of general fully nonlinear second order elliptic equations $F(x, u, Du, D^2u) = 0$ is proved. The case of uniformly elliptic equations as well as degenerate elliptic ones is considered. As an application, existence and uniqueness of the continuous viscosity solutions is obtained. Under the precise conditions for the validity of the comparison principle Lipschitz regularity of the continuous viscosity solutions is given. If these conditions fail, existence of continuous viscosity solutions which are not Lipschitz in the interior of the domain is proved. As a consequence of the interior gradient blow ups the viscosity solutions form interior jump discontinuities with an infinite gradient at the points of discontinuity in the strictly elliptic case. The concept of jump discontinuous viscosity solutions for nonlinear problems with interior gradient blow-up phenomena is discussed.

The minimum time function for control systems with controls in L^p, $p \geq 1$

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We study the properties of the minimum time function to reach the origin and its relative reachable set for the solutions $y_x(\cdot, u)$ of a control system of the form

$$\dot{y}(t) = Ay(t) + Bu(t) \; t > 0, \qquad y(0) = x \in \mathbb{R}^n$$

where the admissible controls belong to the set $U_{p,k} = \{L^p([0, +\infty[, \mathbb{R}^m) : \; \|u\|_p \leq k\}$. Moreover we want to characterize such a function, denoted by $T_p(x, k)$, and the reachable set $R_p = \cup_{k>0}\{x : \exists t \geq 0, \; \exists u \in U_{p,k} : y_x(t, u) = 0\} \times \{k\}$; by means of an opportune boundary value problem associated to the (HJB) equation.

In case $p = +\infty$ everything is well known both for regularity and characterization of T_∞ and R_∞ (see e.g. [1]). If $p > 1$, under controllability conditions there are only regularity results for the k–sections $T_p(\cdot, k)$ (see [2]), and there is no characterization of T_p, R_p through the (HJB) equation. If $p = 1$, (to our knowledge) nothing is known.

As new results we prove that, under the Kalman condition, if $p > 1$ then $(x, k) \mapsto T_p(x, k)$ is continuous on R_p; while if $p = 1$ then T_1 is continuous on the interior of R_1 (R_1 fails in general to be an open set). Moreover for $p \geq 1$ we characterize the function T_p and the reachable set R_p by means of the unique bounded bilateral viscosity solution W_p to the boundary value problem given by

$$\max_{w_0 \geq 0,\ |(w_0,u)|=1} \{-\frac{\partial W}{\partial x}(x,k)(Axw_0 + Bu) + \frac{\partial W}{\partial k}(x,k)|u|^p - e^{-k}w_0^p + W(w_0^p + |u|^p)\} = 0$$

on $(\mathbb{R}^n \setminus \{0\}) \times]0, +\infty[$, and the boundary conditions

$$W(0,k) = 0 \quad \forall k \geq 0, \quad W(x,0) = 1 \quad \forall x \neq 0.$$

More precisely, one obtains that for any fixed $k > 0$ one has

$$T_p(x,k) = -\log(1 - e^k W_p(x,k)); \quad R_p = \{(x,k) : W_p(x,k) < e^{-k}\}$$

if $p > 1$, while for $p = 1$ $\operatorname{Int} R_p = \operatorname{Int}\{(x,k) : W_p(x,k) < e^{-k}\}$.

We remark that these results are proven also for a wide class of nonlinear systems with controls bounded in L^p– norm ($p \geq 1$).

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Existence, uniqueness and regularity of the solution of a Dirichlet problem

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The aim of this lecture is to continue the study begun in [4] and to extend the local regularity properties of the solutions of the elliptic second order divergence form operators to global ones obtaining, for the solution of a Dirichlet problem, the global Hölder regularity.

Let us consider a bounded open set Ω in $\mathbb{R}^n$, $n \geq 3$, and $H_0^{1,p,\lambda}(\Omega)$ the subset of functions $u \in H_0^{1,p}(\Omega)$ such that its first derivatives lie in the Morrey space $L^{p,\lambda}(\Omega)$ and the following Dirichlet problem

$$\begin{cases} \mathcal{L}u \equiv -\sum_{i,j=1}^n (a_{ij}(x)u_{x_i})_{x_j} = div(f) & \text{a. e. in } \Omega, \\ u \in H_0^{1,p,\lambda}(\Omega), & 1 < p < \infty,\ 0 < \lambda < n. \end{cases}$$

The coefficients a_{ij} are discontinuous, precisely they are assumed to be in the subset $V.M.O.$ of the John-Nirenberg space whose elements have norm on the balls vanishing as the radius of the balls approaches zero. The known term $f = (f_1, \ldots, f_n)$ is supposed such that f_i belongs to the Morrey space $\in L^{p,\lambda}(\Omega)$, $\forall i = 1, \ldots, n$.

Under these assumptions there exists the unique solution $u \in H_0^{1,p,\lambda}(\Omega)$ of the above Dirichlet problem.

Moreover, there exists a constant k independent on f such that

$$\|u\|_{C^{0,\alpha}(\bar{\Omega})} \leq k\|f\|_{L^{p,\lambda}(\Omega)}, \text{ where } \alpha = 1 - \frac{n}{p} + \frac{\lambda}{p},\ p > n - \lambda.$$

The technique used is similar to that used for the nondivergence form elliptic equation in [3]. In this paper the authors derive a $C^{1,\alpha}_{loc}$–regularity result, related to that of Caffarelli contained in [1] and [2] about the $W^{2,p}$ viscosity solutions, using an integral representation formula for the second derivatives of the solutions of the above equation.

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Some results on first order Hamilton-Jacobi equations with discontinuous coefficients

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I will discuss Hamilton-Jacobi equations of the form

$$H(x, u, Du) = g(x),$$

where the function g can be discontinuous. It can be shown that viscosity super and subsolutions can be characterized by optimality principles. These principles allow to obtain explicit representation formulas for minimal and maximal solutions, and to prove uniqueness of the viscosity solution under suitable conditions. Examples where the solution is not unique can also be produced.