F. Further topics

6.1 Delay Equations

Nonlinear Retarded Functional Differential Equations of Arbitrary (Fractional) Orders

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The arbitrary (fractional) order integral operator I^α , $\alpha \geq 0$ is a singular integral operator, and the arbitrary (fractional) order differential operator $D^\alpha = I^{-\alpha}$, $\alpha \geq 0$ is a singular integro-differential operator, they generalize (interpolate) the integral and differential operators of integer orders. The fractional calculus $\{I^\alpha \ ; \ \alpha \in R\}$ is enjoying growing interest not only among mathematicians, but also among physicists and engineers. In this lecture we study the existence of monotone solution (in the class $L_1(0,T)$ of Lebesgue integrable functions) of the problems

$$D_a^\alpha \, x(t) \ = \ F(t, x(m(t))) \ , \ \ x(a) \ = \ x_o \ \geq 0 \ , \ \ \alpha \in (0,1],$$

where the function F is a nonlinear function satisfies the Caratheodory conditions, and the more general one

$$D^\alpha x(t) = f(t, x(t), D^{\alpha_1} x(t-r), D^{\alpha_2} x(t-2r)), ., ., ., D^{\alpha_n} x(t-nr)) \ \ , \ t \in I.$$

$$D^j x(t) = 0 \ , \quad for \ \ t \leq 0 \ , \ j = 0, 1, 2, ..., n.$$

Here the function $f(t,U)$ satisfies the Caratheodory conditions i.e., $t \to f(t,U)$ is measurable for every $U \in R^{n+1}$ and $U \to f(t,U)$ is continuous for every $t \in I$ and $\alpha \in (n, n+1]$, $\alpha_k \in (k-1, k]$, $k = 1, 2, ..., n$, and $\alpha_o = 0$.
As special case we also consider the following equations:

$$D^\alpha x(t) = a(t) + \sum_{k=0}^{n} g_k(t, x(t)) D^{\alpha_k} x(t-kr) \ , \ t \in I,$$

$$D^\alpha x(t) = a(t) + \sum_{k=0}^{n} f_k(t, D^{\alpha_k} x(t-kr)) \ , \ t \in I,$$

with the initial conditions (3), where f_k and g_k are nonlinear functions that satisfy the Caratheodory conditions.

Special slowly oscillating periodic solution of a state dependent delay differential equation

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We consider the following state dependent delay differential equation

$$\dot{x}(t) = f(x(t - r(x(t))))$$

where f is a real smooth function. This equation playes an important role in medicine, economy and so forth... We are interested in the existence of slowly oscillating periodic solutions, in particular special symmetric ones.

6.2 Numerics

On the Mathematical Foundation of the Nonstandard Finite Difference Method

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The finite difference method is one of the oldest, simplest and thus very popular technique for the numerical treatment of differential equations. For most of the equations in mathematical physics finite difference schemes have been designed and investigated from both theoretical point of view (convergence) and practical point of view (consistency, stability). Furthermore, provided the solution is smooth enough, these schemes produce numerical solution with optimal rates of convergence when the step-size approaches zero.

One disadvantage of this standard approach is that qualitative properties of the exact solution are not transferred to the numerical solution. In practice, the limit of the step-size is not reached. What we have is the numerical solution obtained for one or several values of the step-size. Thus, the stated disadvantage might be catastrophic.

The nonstandard approach discussed in this talk preserves essential properties of the exact solution. This is achieved by replacing derivatives by nonstandard finite difference operators with denominators that are suitable function of the step-size, nonlinear expressions being, in general, approximated nonlocally.

Nonstandard finite difference techniques were developed empirically [1] for solving practical problems in applied sciences and engineering. Although they produce results satisfactory to their users, these techniques have not yet been subjected to rigorous mathematical analysis. Some concepts are still unclear.

Our aim is to give rigorous mathematical meaning and justification of some key concepts involved in the design of nonstandard finite difference schemes. We also discuss a certain number of "successful" empiric procedures the mathematical justification of which is still pending.

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Optimal derivative procedure of a nonliear ODE.

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We introduce a certain notion of approximation defined as the optimal derivative of the following nonlinear ordinary differential equation

$$\frac{dx}{dt} = F\left(x\left(t\right)\right), x\left(0\right) = x_0.$$

This approximation is global in the neighborhood of a steady state. That is obtained as a sort of mean value of the derivative of the nonlinear function along trajectories linking the initial value to the origin. The procedure is based on the minimization of a certain functional with respect to a curve starting from an initial value x_0 and going to 0 as t goes to infinity. At each step, it gives a linear map, starting from the Jacobian matrix $DF\left(x\right)$ estimated at the initial value x_0. The optimal Derivation of the nonlinear equation is obtained as a limit of the sequence of linear maps determined by the procedure and can be written as

$$\frac{dx}{dt} = \tilde{A}x\left(t\right), x\left(0\right) = x_0.$$

Our results are in the line of previous work by Vujanovic in (1973) and Jordan et al (1987).

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An Approximate Method of Solution of Some Types of Singular Integro-Differential Equations

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The presentation is devoted to approximate method of solutions of singular integro-differential equations, which is based on the approximation of functions by interpolated polynomials, fundamental functions of which are characteristic functions of differential and integral operators. This method was discussed in the papers [1]. We here spread it on the Cauchy problem for many types of singular and hypersingular integro-differential equations as:

$$a(t)x(t) + b(t)\int_{-1}^{1} \frac{p(\tau)x'(\tau)d\tau}{\tau - t} + \lambda \int_{-1}^{1} p(\tau)h(\tau)d\tau = f(t),\ -1 < t < 1,$$

where $p(t)$ is a weight, $a(t), b(t), h(t), f(t) \in H_\alpha$,

$$\int_{-1}^{1} \frac{p(\tau)x'(\tau)d\tau}{(\tau - t)^2} + \lambda \int_{-1}^{1} p(\tau)h(\tau)d\tau = f(t),\ -1 < t < 1,$$

where $\int_{-1}^{1} x(t)t^{-2}dt$ is hypersingular integral.

For example, let us consider the Cauchy problem for the Prandtl singular integro-differential equation of finite wing:

$$K\Gamma \equiv \frac{\Gamma(t)}{b(t)} - \frac{1}{\pi}\int_{-1}^{1} \frac{\Gamma'(\tau)d\tau}{\tau - t} = f(t), \quad \Gamma(-1) = \Gamma(1) = 0$$

where $-1 < t < 1$, $b(t), f(t) \in H_\gamma$ $(0 < \gamma < 1)$. We obtain the approximate solution in the analytical form

$$\Gamma_n(t) = \sqrt{1 - t^2}\sum_{k=0}^{n}\left(\frac{1}{\gamma_k}\sum_{i=0}^{n} U_i(\mu_k)U_i(t)\right)\frac{f(\mu_k)B(\mu_k)}{(\sqrt{1 - \mu_k^2} + B(\mu_k))}$$

where $\gamma_k = \sum_{i=0}^{n} U_i^2(\mu_k)$ is a Chebyshev polynomial of the second kind, k is its degree and μ_k, $k = 0, 1, \ldots, n$, are the roots of the polynomial $U_{n+1}(t)$.

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Event recognition using a binary discriminant tree

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In this paper, we propose a new design procedure for constructing a multivariate and discriminant binary tree for classifying high energy reactions data.

A binary decision tree is usually generated by repeated splits of the learning event set into two descendant subsets [1]. At each node of the tree, the split is obtained by a cut on the selected variable. Several attempts have been made to construct binary trees otherwise [2-4].

Instead of an univariate design, we suggest to use some linear discriminant functions. These discriminant functions are combinations of discriminant variables. Their choice is imposed by the topology of the events to be classified. Moreover, the computation of the functions' cuts values is based on minimizing the Kolmogorov- Smirnoff distance. Finally, a test of performance using efficiencies and purities of classifications is used.

Experiments on real and simulated test data, containing four classes of high energy reactions events show that the proposed discriminant tree classifier is faster and more efficient than the univariate binary decision tree.

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On the investigation of boundary value problems

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One of the problems which will be considered is the following two-point non-linear boundary value problem:

$$\begin{aligned} dx/dt &= f(t,x,\lambda_1,\lambda_2),\\ Ax(0)+C(\lambda_1)x(T) &= d(\lambda_1,\lambda_2), \qquad x_1(0)=x_{10},\ x_2(0)=x_{20}. \end{aligned}$$

Here, $f \in C([0,T]\times D\times[a_1,b_1]\times[a_2,b_2],R^n)$, $D\subset R^n$ $(n\geq 3)$ is closed and connected, and $d\in C([a_1,b_1]\times[a_2,b_2],R^n)$; A and $C(\lambda_1)$ are members of $R^{n\times n}$ such that $\det[k_1A+k_2C(\lambda_1)]\neq 0$ for some real $k_1\neq k_2$ and all $\lambda_1\in[a,b]$; $\lambda_1\in[a_1,b_1]$ and $\lambda_2\in[a_2,b_2]$ are unknown scalar parameters.

We give a numerical-analytic method of successive approximations to establish existence theorems for a solution $(x^*,\lambda_1^*,\lambda_2^*)$ and construct approximations to the solution.

It is shown that, under certain conditions, the sequence

$$\begin{aligned} x_{m+1}(t,z,\lambda) &= z+k_1Hd_1(z,\lambda)+\int_0^t f(s,x_m(s,z,\lambda),\lambda_1,\lambda_2)ds\\ &-\frac{t}{T}\int_0^T f(\tau,x_m(\tau,z,\lambda),\lambda_1,\lambda_2)d\tau+\frac{t}{T}Hd_1(z,\lambda)ds, \end{aligned}$$

where $m=0,1,2,\ldots$, $x_0(t,z,\lambda)\equiv z+k_1Hd_1(z,\lambda)$, $H=[k_1A+k_2C(\lambda_1)]^{-1}$, $d_1(z,\lambda)=d(\lambda_1,\lambda_2)-[A+C(\lambda_1])z$, $\lambda=(\lambda_1,\lambda_2)$,

$$z=\mathrm{col}(x_{10},x_{20},y_1,\ldots,y_{n-2})=\mathrm{col}(x_{10},x_{20},y),$$

uniformly converges.

The function $x^*(t,z,\lambda) = \lim_{m\to\infty} x_m(t,z,\lambda)$ is a solution of the original problem iff the pair (y,λ) satisfies the determining equation

$$\Delta(y,\lambda) := \frac{1}{T}(k_2-k_1)Hd_1(z,\lambda) - \frac{1}{T}\int_0^T f(t,x^*(t,z,\lambda),\lambda_1)dt = 0.$$

A survey of the investigations concerning the method under consideration for various types of problems can be found in [1, 2].

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Coupling of kinetic and hydrodynamic equations: the case of Carleman model

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The coupling of kinetic equations and their hydrodynamic limits was introduced and studied by Le Tallec and Tidriri [2,3] and Tidriri [4]. This approach was introduced in order to solve several difficulties that occur at the interface between fluid mechanics and kinetic theory. Because of the practical importance of these methodologies, the establishment of their mathematical foundations is of crucial importance. The mathematical theory of such coupling started in [4] and continued in [2,3], where the coupling of two models of hydrodynamical type is considered. In [5,7], the author provided an analysis of the coupling of two models of kinetic type. In [6] the author provided the analysis of the coupling of kinetics equations and their hydrodynamics limits for the Carleman model. In these papers, the author established the existence theory and the asymptotic behaviour of the resulting coupled systems. Moreover, he studied the semi-discretized (with respect to the time variable) coupled systems via the Time Marching Algorithm also introduced in [2,4] and established the convergence theory for the resulting coupled problems. Finally, he provided numerical results confirming the above mentioned mathematical results.

As far as we know the above mentioned results are the first mathematical results about the analysis of coupled systems of kinetic equations or coupled systems of kinetic equations and their hydrodynamical limits.

In our proposed talk, we shall first, present a motivation to our studies by giving some numerical results and experimental evidences that show why such coupling is of crucial practical importance. we shall then give more details about the analysis of the coupling of kinetic equations and their hydrodynamics limits.

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6.3 Ordinary Differential Equations

Uniformly Isochronous Quintic Planar Vector Fields

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In this paper we analyze the center problem, and consequently the isochronicity, for the vector fields $\left(-y + x\sum_{i=1}^{4} H_i(x,y), x + y\sum_{i=1}^{4} H_i(x,y)\right)$, with H_i homogeneous polynomial of the i-th degree, $i = 1, \ldots, 4$, and $H_iH_4 \neq 0$ for some $i = 1$, 2 or 3. To treat such problem basically we use procedure based in normal forms and also characterization of center through the existence of Lie's symmetries of the system (see [**1**]). Besides, we give commutators, null divergence factors and first integrals of some subfamilies. We also include the phase portrait of the centers with polynomial commutators on the Poincare disk. We finish the paper with two open problems.

The vector fields we are studing belong to the family

$$(S) \qquad (-y + xH(x,y), x + yH(x,y)),$$

where H is an analitic function in a neighbourhood of the origin, with $H(0,0) = 0$. As far as we know, some subfamilies of these systems have been studied by several authors. [**4**] analyzes (S) with H an homogeneous polynomial of an arbitrary degree, drawing to the conclusion that if the degree of H is odd the origin is a center, whereas if it is even it's necesary that the integral of H on S^1 must vanish. Moreover, in [**1**] the isochronous condition have been obtained according to the coeficients of the systems. [**7**] research some properties for first integrals and for commutators of the systems studied by Conti with H an homogeneous function. [**6**] provides linearizating changes for the reversibles systems (S) with $H = H_1 + H_2$. Next, [**5**] removes the reversibility condition and shows that the only existing centers are the reversible ones. [**2**] and [**3**] study, in an independent way, the centers of the systems (S) with $H = H_1+H_2+H_3$, with $H_iH_3 \neq 0$, $i = 1, 2$. All of them are reversible centers and there are some isochronous ones which don't have polynomial commutator.

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Investigation of Resonances Using Higher Order Normal Forms

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In the present analysis the higher order normal forms technique [2] is applied to the local analysis of the shock adiabat in an elasto-plastic solid and the transient resonance response of a non-linear slow-variant simple degree of freedom system. The solution of these problems is reduced to the investigation of a trajectory dynamic which is governed by the corresponding systems of non-linear ordinary differential equations. The local analysis in the vicinity of a stationary point is provided in the resonance case when the corresponding linear part has a multiple zero eigenvalue. The step to step technique of nearly identity transformations is applied to the analysis of a dynamics on the central manifold.

The shock adiabat in elasto-plastic solid with the smooth convex yield surface is constructed by embedding the Kelvin-Voigt type viscosity element in the rheological model [1], [3]. In the general case of a combined loading in the viscous layer the problem is reduced to the investigation of the following system of ordinary differential equations

$$\frac{du_i}{d\xi} = \Big(L_{1i1p}(\sigma_{\alpha\beta}) - (c^2+\varepsilon)\delta_{ip}\Big)u_p\,, \qquad \frac{d\sigma_{ij}}{d\xi} = -L_{ijk1}(\sigma_{\alpha\beta})u_k$$

here ξ is the coordinate of a viscous layer, u_i and $\sigma_{\alpha\beta}$ are accelerations and stresses, $L_{ijkl}(\sigma_{\alpha\beta})$ are components of the acoustic tensor, c is the sound velocity at the state after the shock, ε is the small nonnegative parameter.

Small intensity shock waves correspond to the heteroclinic type trajectories

$$\lim_{\xi\to\pm\infty} u_i(\xi) = 0\,, \quad \lim_{\xi\to\pm\infty} \sigma_{ij}(\xi) = \sigma_{ij}^{\pm}$$

which satisfy the inequality of an active plastic load:

$$A_{pqk1}F_{pq}(\sigma_{\alpha\beta})u_k \leq 0$$

A_{ijkl} are elastic moduli, $F_{ij}(\sigma_{\alpha\beta})$ is the yield surface gradient.

Using the Lax evolutionary conditions the types of bifurcations with respect to a change in the velocity of a shock wave are found. An integration of the normal form is reduced to the investigation of an ordinary differential equation of the first order. The conditions of solvability which follow from the inequality of active plastic load are established. Numerical simulations of heteroclinic orbits are performed.

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Differential Equations of Infinite Order with Subordinate Terms

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Let $G \subset R^{\nu}$, $\nu > 1$, be a bounded domain and Γ be a boundary of the domain. The solvability of the following Dirichlet problem for nonlinear differential equations of infinite order is being considered

$$L_1(u) + L_2(u) = \sum_{|\alpha|=0}^{\infty} (-1)^{|\alpha|} D^{\alpha} A_{\alpha}(x, D^{\gamma}u) + \sum_{|\alpha|=0}^{\infty} (-1)^{|\alpha|} D^{\alpha} B_{\alpha}(x, D^{\omega}u) = h(x),$$

$$x \in G, \tag{1}$$

$$D^{\beta}u(x) = 0, \qquad x \in \Gamma, \qquad |\beta| = 0, 1, \ldots . \tag{2}$$

The $A_\alpha(x, D^\gamma u)$, $B_\alpha(x, D^\omega u)$ are continuous functions of $x \in G$ and of all possible derivatives $D^\gamma u$ and $D^\omega u$, $|\gamma| \leq |\alpha|$, $|\omega| \leq |\alpha|$. The right side $h(x)$ of equation (1) may be a generalized function with singularity of infinite order.

Definition. The differential operator $L_1(u)$ is called principal in comparison with operator $L_2(u)$ (henceforth referred to as subordinate) if the space, being the domain of $L_1(u)$, is compactly embedded in the space corresponding to $L_2(u)$.

The conditions for compact embedding of the corresponding spaces for the comparison of the operators $L_1(u)$ and $L_2(u)$ has been studied. That allowed us to receive the conditions of solvability of the problem (1), (2), to indicate the methods of its examination. If (1) is the linear equation containing subordinate terms then it is Fredholm. That theory has allowed to receive the solvability of the series of problems which could not be solved earlier. That is demonstrated by examples.

On n-dimensional Riccati extended forms

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We deal with initial value problems for the system of implicit difference equations of the form

$$A^*(n)\Phi(n) - \Phi(n+1)f^*A^*(n)\Phi(n) = 0, \qquad n = 0, 1, \dots,$$

where the $(m+1)\times(m+1)$ complex matrices $A(n)$ are composed of entries $a_{i,j}(n)$, $i, j = 1, \dots, m+1$, f is a fixed nonzero vector of $m+1$ complex components. The initial value is prescribed as $\Phi(0) = \Phi_0$.

For the lowest dimension ($m = 1$), the problem reduces to iteration

$$x\,(n+1) = \frac{\overline{a}_{11}\,(n)\,x\,(n) + \overline{a}_{12}\,(n)}{\overline{a}_{21}\,(n)\,x\,(n) + \overline{a}_{22}\,(n)}, \quad n = 0, 1, \dots.$$

This iteration with constant real coefficients was studied by Brandt. Later, Kočic and Ladas considered real varying coefficients, too. Recently in the joint paper with Agarwal, we investigated the case with complex, constant coefficients when $m = 2$. We described the equilibria. Some problems of asymptotic stability were addressed there, too.

In general case, we observe that there exists at most one local solution $\Phi(n)$, $n = 0, \dots, N+1$, of the problem such that $f^*A^*\,(n)\,\Phi\,(n) \neq 0$, $n = 0, \dots, N$. For this local solution $f^*\Phi\,(n) = 1$, $n = 1, \dots, N+1$, holds.

It turns out that the local solution can be given directly. It helps us to formulate a criterion for the the existence of the global solution.

Attention is paid to the constant case. The results obtained for the case $m = 2$ earlier for equilibria and asymptotic stability are extended now, as well.

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On the Solutions of the Second Order Differential Equation with Variable Parameters

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Consider a linear differential equation

$$\frac{d^2x}{dt^2} + p(t)\frac{dx}{dt} + q(t)x = 0, \tag{1}$$

where t is time, $p(t), q(t)$ are continuous functions which are bounded on every compact. In [1] one can find that no solution of (1) can possess a zero of order greater than one. Let t_1 and t_2 denote consecutive zeros of some solution $x(t)$ of the equation (1) $(t_2 > t_1)$.

Theorem. Let $a = \sup\limits_{t_1 \le t \le t_2} |p(t)|,\ b = \sup\limits_{t_1 \le t \le t_2} |q(t)|$. Then

1) $t_2 - t_1 \ge \frac{4}{\sqrt{4b-a^2}}(\frac{\pi}{2} - \arctan\frac{a}{\sqrt{4b-a^2}})$ if $4b > a^2$;

2) $t_2 - t_1 \ge \frac{4}{a}$ if $4b = a^2$;

3) $t_2 - t_1 \ge \frac{2}{\sqrt{a^2-4b}} \ln\frac{a+\sqrt{a^2-4b}}{a-\sqrt{a^2-4b}}$ if $4b < a^2$.

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Some Criteria on the Uniqueness of Limit Cycles in planar Polynomial Systems

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In this article we give criteria on the uniqueness of limit cycles in planar polynomial systems. It is well known that provided a system has an invariant straight line, there can be at most one limit cycle [1]. Also that parabolas, which are particular solutions of quadratic systems can coexist with limit cycles [2]. We prove the uniqueness of some known algebraic limit cycles for quadratic systems. We present also criteria and proofs on the existence and uniqueness of limit cycles under some conditions, as well as open questions on this subject.

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Some unexpected properties of limit cycles of quadratic systems in the plain

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In the generic case every autonomous quadratic system in the plane, which has a finite singular point, can be reduced to a quadratic system with five essential parameters. We consider two of them

$$\frac{dx}{dt} = -(x+1)y + \alpha Q(x,y) \ , \quad \frac{dy}{dt} = Q(x,y) \ ,$$
$$Q = x + \lambda y + ax^2 + b(x+1)y + cy^2 \ ; \tag{1}$$

$$\frac{dx}{dt} = 1 + xy \ , \quad \frac{dy}{dt} = \sum_{i+j=0}^{2} a_{ij} x^i y^j \ , \quad \sum_{i+j=0}^{2} a_{ij}(-1)^j = 0. \tag{2}$$

System (1) has the larger number of rotation parameters λ, b, α. For every parameter of system (1) (for example, b) we can define the curve of limit cycles at every point (x, b) on the plane $x = b$ which corresponds to a limit cycle of system (1) passing through the point $(x, 0)$ for given value parameter b. The curve of limit cycles corresponds to a manifold of limit cycles in the space (x, y, b). It is known [1] that for a given rotation parameter (for example, b) the curve of limit cycles corresponds to a unique function of the variable x having a point of Andronov-Hopf bifurcation. We establish that in contrary for parameters a, c corresponding curves of limit cycles can have some branches without points of Andronov-Hopf bifurcation and they can have also turning points. We establish that quadratic system (2) can have around a weak focus $A(1, -1)$ a separatrix cycle with two saddles at infinity and critical stability ($a_n = 0$).

We prove also that it is possible to take as Dulac's function the function $B = |\Psi(x,y)|^{1/k}$, $k = const$, Ψ is a polynomial with respect to y of degree $2m$, such that

$$div(Bf) = \frac{1}{k} sign\Psi \, |\Psi|^{1/k-1} P(x), P(x) > 0, x > 0.$$

It means [2], that the number of limit cycle of system (2) around the focus $A(-1,1)$ is not more than m, $m = 1,2,3$; f is the vector field associated with system (2). So we have a posibility to attack the Hilbert problem for quadratic systems.

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Limit sets in planar systems of piece-wise linear differential equations with a line of discontinuity

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We consider planar systems of piece-wise linear differential equations with a line of discontinuity. Systems of this type have applications in science and engineering. They provide mathematical models for mechanical systems with Coulomb friction, or for electrical circuits with a thin triode, or for direct control systems with a two-point relay characteristic, see [1]. We estimate all possible limit sets of such systems. They consist of stationary, periodic, homoclinic and heteroclinic orbits. We study the existence, number and stability of the orbits mentioned above. In our analysis we distinguish between stationary points, which do or do not belong to the line of discontinuity, and orbits with or without sliding motion, see [2].

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On stability and bifurcations in nonlinear systems of differential equations

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We consider the nonlinear system of differential equations

$$\frac{dx}{dt} = A(\mu)x + X(\mu, x), \;\; X(\mu, 0) = dX(\mu, 0) = 0 \tag{1}$$

continuously depending on a vector parameter μ.

One of the most studied phenomena arising in the neighborhood of the point $x = 0$ is the Hopf's bifurcation. It arises when by $\mu = 0$ the matrix A(0) has a pair of pure imaginary eigenvalues. In this talk the analogous problem connected with the study of higher codimension bifurcation is considered.

Our main concern was with the following problems.

i) Stability of the equilibrium point of the system (1), when at $\mu = 0$ the matrix $A(0)$ has m a pair of purely imaginary eigenvalues. (The results obtained for the problem (i) relate to two different situations: non-resonance case and resonant case.)

ii) Bifurcation of m-dimensional invariant tory, when $\mu \neq 0$ (non-resonance case).

iii) Bifurcation of steady resonance modes.

Lyapunov functions in estimates of dimension of attractors of dynamical systems

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Lyapunov functions are introduced into the estimates of Hausdorff and fractal measures of the sets which are shifted along the trajectories of differential equations. As a result new estimates of attractors also containing Lyapunov functions are obtained. Lyapunov functions in the estimates of Hausdorff dimension of attractors of Lorenz system are constructed. With the help of these functions the validity of Eden conjecture that the dimension of Lorenz attractor is estimated by the Lyapunov dimension of a zero saddle point is proved.

For linear homogeneous continuous and discrete systems frequency-domain estimates of singular numbers of fundamental matrices are received. Here Yakubovich-Kalman and Kalman-Szego theorems about solvability of matrix inequalities are applied. On the base of frequency-domain estimates of singular numbers estimates of Lyapunov dimension of Lorenz and Hennon attractors are obtained. The comparison of these estimates with computer experiments is done.

The connection of these results and the classical theory of motion is discussed. Special attention is paid to the global stability.

On the Stability of a Second Order Linear Differential Equation with Variable Parameters

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Consider an oscillator described by the following differential equation

$$\ddot{x} + f(t)\dot{x} + g(t)x = 0 \tag{1}$$

where the damping and rigidity coefficients $f(t)$ and $g(t)$ are continuous and bounded functions of the time t. Most of the theories examining a stability problem of the zero solution are based on the Lyapunov stability and instability theorems and the corresponding Lyapunov function is assumed as an energy-type function

$$V = \frac{1}{2}c_1(t)\dot{x}^2 + \frac{1}{2}c_2(t)x^2$$

where $c_1(t), c_2(t)$ are time variable functions. In [1], A.P.Merkin considered the case $c_1(t) = c_2(t) = 1$ and stability conditions were obtained only for constant f and g. An extension was done in [2] for periodic functions $f(t)$ and $g(t)$. By means of a Lyapunov function which is a quadratic form with respect to x and $\dot{x}$, V.M.Starzhinsky [3] (assuming that $0 < l \le f(t) \le L,\ 0 < m \le g(t) \le M$) obtained sufficient conditions of asymptotical stability for the solution

$$x = 0, \quad \dot{x} = 0 \tag{2}$$

of equation (1). They are written as restrictions to the constants l, L, m, M.

In this paper sufficient asymptotic stability conditions of the solution (2) are obtained which are close to necessary and sufficient conditions of stability. We suppose that $g(t)$ is continuously differentiable and that the inequalities

$$|f(t)| < M_1, \quad |g(t)| < M_2, \quad |\dot{g}(t)| < M_3$$

hold for $t \in R_+ = [0; \infty)$.

Theorem. If the conditions

$$g(t) > \alpha_1 > 0, \qquad p(t) = \frac{1}{2}\frac{\dot{g}(t)}{g(t)} + f(t) > \alpha_2 > 0$$

are fulfilled, then the solution (2) of the differential equation (1) is uniformly asymptotically stable.

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Decoupling of impulsive differential equations

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Impulsive differential equations provide an adequate mathematical models of evolutionary processes that suddenly change their state at certain moments. The theme of this communication is the dynamical equivalence of impulsive differential system to a system which is simpler than the given one in terms of decoupling and linearization.

Let us consider the impulsive system in the Banach space $\mathbf{U} = \mathbf{X} \times \mathbf{Y}$

$$\left\{ \begin{array}{l} dx/dt = A(t)x + f(t,u),\ dy/dt = B(t)y + g(t,u), \\ \Delta x|_{t=\tau_i} = x(\tau_i + 0) - x(\tau_i - 0) = C_i x(\tau_i - 0) + p_i(u(\tau_i - 0)), \\ \Delta y|_{t=\tau_i} = y(\tau_i + 0) - y(\tau_i - 0) = D_i y(\tau_i - 0) + q_i(u(\tau_i - 0)), \end{array} \right. \tag{1}$$

where $u = (x, y)$, $i \in \mathbb{N}$, f, g, p_i, q_i satisfy Lipschitz conditions with small ε uniformly with respect to t. Let $\Phi(\cdot, t_0, u_0)\colon \mathbb{R} \to \mathbf{U}$ be the solution of the system (1), where $\Phi(t_0 + 0, t_0, u_0) = u_0$.

Definition. Two impulsive differential systems are *dynamically equivalent* if there exists a map $H\colon \mathbb{R} \times \mathbf{U} \to \mathbf{U}$ and function $e\colon \mathbf{U} \to \mathbb{R}_+$ such that
(i) $H(t, \cdot)\colon \mathbf{U} \to \mathbf{U}$ is a homeomorphism.
(ii) $H(t, \Phi(t, t_0, u_0)) = \Psi(t, t_0, H(t_0, u_0))$ for all $t \in \mathbb{R}$, where $\Psi(\cdot, t_0, u_0)\colon \mathbb{R} \to \mathbf{U}$ is the solution of the second system.
(iii) $\max\{|H(t,u) - u|, |H^{-1}(t,u) - u|\} \leq e(u)$.

We find sufficient conditions under which the impulsive system (1) is dynamically equivalent to the system

$$\left\{ \begin{array}{l} dx/dt = A(t)x + f(t, x, k(t,x)),\ dy/dt = B(t)y, \\ \Delta x|_{t=\tau_i} = C_i x(\tau_i - 0) + p_i(x(\tau_i - 0), k(\tau_i - 0, x(\tau_i - 0))), \\ \Delta y|_{t=\tau_i} = D_i y(\tau_i - 0). \end{array} \right. \tag{2}$$

The last system splits into two parts. The first of them does not contain the variable y, while the second one is linear. This result allows one to replace the given system by a simpler one. Relevant results concerning decoupling and simplifying of the impulsive differential systems are given also. The proof of the theorems is based on a Green maps and a functional integral equations technique.

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Autonomous Differential Systems with Prescribed Limit Set

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We present a class of non-linear differential systems for which an invariant set can be prescribed. Moreover, we show that a system in this class can be explicitly solved if a certain associated linear homogeneuos system can be solved. As a simple application we construct a plane autonomous system having a given closed curve on polar form as the only limit cycle .

Monotonicity and boundedness of the atomic radius in the Thomas-Fermi theory

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Consider an atom consisting of N electrons and a nucleus of positive charges Z. The ground state density of electrons is a non negative function $\rho \in L^1(\mathbb{R}^3)$ such that $\int_{\mathbb{R}^3} \rho(x)dx = N$. In the semiclassical Thomas-Fermi approximation the consistency for a neutral atom $(N = Z)$ can be expressed : ϕ denoting the total electric potential,

(A1) $\quad \phi(x) = Z\ |x|^{-1} - \int \rho(y)\ |x-y|^{-1}\ dy$ (Electrostatic attraction and repulsion) and

(A2) $\quad \rho(x) = [2\sqrt{2}/(3\pi^2)]\ \phi(x)^{3/2}$ (Pauli-principle).

For any $k > 0$, the radius $R_k(N,Z)$ such that k electrons lie outside $B(0, R_k(N,Z))$ is defined by

(A3) $\quad \int_{|x|>R_k(Z)} \rho(x)dx = k; \quad R_k(Z) := R_k(N,Z)$.([1])

After some normalizations using the Thomas-Fermi scaled potential ($\phi^{TF}(x) := Z^{4/3}u(z^{1/3}x)$), these equations are rewritten as:

$u'' + \frac{2}{r}u' = u_+^{\gamma}, \quad r > 0; \quad \gamma = 3/2$

$\rho(r) = u(r)_+^{3/2} \quad \int_{r>R(z)} \{z^{4/3}\ u(z^{1/3}\ r)\}^{3/2} r^2 dr = k.$

After some qualitative investigations of a more general (singular ground state) problem

$$U'' + \frac{a}{r}U' = U_+^{\gamma}, \quad r > 0; \quad a > 1; \quad \lim_{r \searrow 0} U(r) = +\infty; \quad \lim_{r \nearrow \infty} U(r) = 0 \qquad (1)$$

we prove the following main theorem for $a = 2, \quad \gamma = 3/2$ and any $k > 0$:

Theorem A

The problem (S0) has a unique solution $U \in C^2((0,\ \infty))$ such that $U \simeq r^{1-a}$ at 0

and $U \simeq r^{-4}$ at ∞ and
i) any admissible $z \equiv Z$ in (A1) - (A3) satisfies
$z \geq z_k := k/\{\int_0^\infty U(r)^{3/2}\, r^2 dr\}$;
ii) $R_k(z_k) = 0$ and $\{d/dz\}R_k(z) > 0$ in $z \geq z_k$;
iii) $0 < \lim_{z \nearrow \infty} R_k(z) := R_\infty < \infty$.

The monotonicity established here of the radius in Thomas-Fermi theory agrees with the experimental fact that atomes grow in size as one goes down each group in the periodic table. (The radius does not increase with Z throughout the whole periodic table but if one considers a fixed group (e.g. the noble gases), it does).

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6.4 Partial Differential Equations

On the Dirichlet Problem for the Nonlinear Parabolic Equations in Non-smooth Domains

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We study the Dirichlet problem for the parabolic equation

$$u_t = a\Delta u^m + b \bigtriangledown \cdot u^\gamma + cu^\beta$$

with $a, m, \gamma, \beta > 0, b, c \in I\!R$, in a bounded, non-cylindrical and non-smooth domain $\Omega \subset I\!R^{N+1}, N \geq 2$. Existence and boundary regularity results are established. We introduce a notion of parabolic modulus of left-lower (or left-upper) semincontinuity at the points of the lateral boundary manifold and show that the upper (or lower) Hölder condition on it plays a crucial role for the boundary continuity of the constructed solution. The Hölder exponent $\frac{1}{2}$ is critical as in the classical theory of the one-dimensional heat equation $u_t = u_{xx}$.

Problem of Small Denominators in the Theory of Continuous Systems Nonlinear Oscillations

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The problem of nonlinear oscillations of continuous systems is an outstanding one in science and engineering and has received considerable attention in recent years. From a fundamental, as well as design, standpoint it is desirable to calculate periodic solutions in an analytical way. Unfortunately, an exact determination of periodic solutions of nonlinear continuous systems is generally out of the question, even for the simplest case (nonlinear wave equation). The most natural ways for approximate solving of problem under consideration are variational and asymptotic approaches. Here must be noted papers of P.Rabinovitch, H.Bresis (variational approaches), G.Wayne, J.Bourgain, W.Craig (KAM theory), Yu.A.Mitropol'sky et al. (averaging), S.L.Lau et al. (multyscale method). In our paper we propose and analyze two new asymptotic approaches for construction of periodic solutions of nonlinear PDE. The difficulty in the construction is that one must overcome the phenomenon of small denominators that is present in this problem. We think that these novel approaches to small denominators problems are robust, and give an interesting alternative to the classical approach of the KAM theory.

Our first approach is based on the so-called "small δ method", proposed by C.M.Milton et al. The main idea of this approach may be described as follows. Let us consider boundary value problem for nonlinear wave equation

$$u_{tt} = u_{xx} - \varepsilon u^3, \tag{1}$$

$$u\left(0,t\right) = u\left(\pi,t\right) = 0. \tag{2}$$

We introduce artificial "small parameter δ" in such a way, that

$$u^3 = u^{1+2\delta} = u\left(1 + \delta \ln\left(u^2\right) + 0.5\delta^2\left(\ln\left(u^2\right)\right)^2 + ...\right). \tag{3}$$

Then we represent solution of boundary value problem (1), (2) in the form of expansion

$$u = u_0 + \delta u_1 + \delta^2 u_2 + ... \tag{4}$$

It is very important that thanks expansion (3) we have only one resonance term in each approximation, and routine Poincaré-Lindstedt technique may be used for it elimination. In our solution we must not suppose parameter small, and we may construct periodic as well as quasi-periodic solutions.

Our second approach is valid for small value of ε and is based on the renormalization procedure with artificial "small parameter" introduction. First of all from the conditions of secular terms absence we obtain infinite system of nonlinear algebraic equations. Then we introduce artificial "small parameter μ" in order to diagonalize this system, and we search solution in the form of expansions in powers of μ. Then we use Padé approximants for truncated series in μ and set $\mu = 1$. Proposed approaches are in some sense complimentary to each other and may be greatly extended.

Chaotic eigenfunctions, maximum norms and random superpositions of plane waves

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The growth of the maximum norms of quantum eigenstates of classically chaotic systems with increasing energy is investigated. The maximum norms provide a measure for localization effects in eigenfunctions. An upper bound for the maxima of random superpositions of random functions is derived. For the random-wave model this gives the bound $c\sqrt{\ln E}$ in the semiclassical limit $E \to \infty$. The growth of the maximum norms of random waves is compared with the growth of the maximum norms of the eigenstates of six quantum billiards which are classically chaotic. The maximum norms of these systems are numerically shown to be conform with the random-wave model. Furthermore, the distribution of the locations of the maximum norms is discussed.

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On differential operators of Mizohata type

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The differential operator of Mizohata

$$\frac{\partial}{\partial t} + it^{2n+1}\frac{\partial}{\partial x}$$

is not locally solvable at the origin of R^2. We call differential operators of Mizohata type the next operators

$$M = \frac{\partial}{\partial t} + ib\,(t)\,\frac{\partial}{\partial x}$$

where $b(t)$ is a real function of class C^∞, satisfying $tb(t) > 0$,$\forall\ t \in R^*$. These operators are also not locally solvable at the origin , because they do not satisfy the Nirenberg-Treves condition (P) ([3])

In this work we study the differential equation

$$Mu = f$$

where f is a regular function in R^2 .

We give necessary and sufficient conditions on f such that this last equation has a C^1 solution in a neighbourhood of the origin. These results generalise the results obtained in the case of the Mizohata equation by other authors (see [4], [1], [2]).

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Goursat problems, multiparameter elliptic operators and related spectral asymptotics

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Partial differential equations with multidimensional time occur in various bran- ches of science, for example, in the theory of probability and Brownian motion, the theory of boundary layers, cosmology and physics.

We introduce a new class of polyparabolic equations with multidimensional time and develop the existence and uniqueness theory for Goursat like problems in the corresponding Sobolev-like scale of spaces.

On our way to the existence and uniqueness theorem for polyparabolic problems we study elliptic boundary value problems with parameters in a bounded domain.

The notion of the ellipticity with parameters for differential and pseudodifferential operator pencils is applied to the resolvent construction, which leads to the definition of the corresponding complex powers and $\zeta-$function, i.e. to a new functional calculus of operators with promising applications to multiparameter spectral theory.

We prove the extension theoremfor the traces of the kernels of the complex powers and the corresponding $\zeta-$function. The last appears to be a meromorphic function in C^m (m is the number of parameters) with polar sets of the first order and the corresponding holomorphic residue-forms.

In particular, for $m = 2$, this investigation gives the opportunity to write a two-parameter spectral asymptotics in terms of these forms using the two-dimensional refinement of the Ikehara Tauberian theorem.

Mathematical analysis of the finite elasticity BVP for materials with ideal saturation

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For materials with ideal saturation the elastic potential $\Phi(x, \nabla u)$ of which has the linear growth in $|\nabla u|$, where $x \mapsto u \in R^3$ is the map, the existence of the limited static load (such external static forces with no solution of BVP) and discontinuous maps with breaks of the sliding type was proved by the author in [1,2]. From the physical point of view these effects are treated as the destruction of material.

For estimation of the mechanical durability of the elastic solid $\Omega \in R^3$ the following variational problem must be solved

$$t_* = \inf \left\{ \int_\Omega |\nabla u(x)| \lambda(x)\, dx : \ u \in V, \ L(u) = 1 \right\}, \tag{1}$$

$$L(u) = \int_\Omega \langle f, u \rangle\, dx + \int_{\Gamma^2} \langle F, u \rangle\, d\gamma, \qquad \langle g, u \rangle(x) = \int_x^{u(x)} g(x, u) \cdot du,$$

$$\lambda(x) = \inf \left\{ \frac{\Phi(x, Q)}{|Q|} : \ Q \in R^{3\times 3} \right\},$$

where $V = \{u \in W^{1,1}(\Omega, R^3) : \ u(x) = u^0(x), \ x \in \Gamma^1\}$ is the set of admissible maps, u^0 is the given map of a portion Γ^1 of the boundary $\partial\Omega$, f and F are the external mass and surface forces, respectively, and $\Gamma^2 = \partial\Omega \setminus \Gamma^1$. The limited static load is $(f^*, F^*) = t_*(f, F)$. If $t_* < 1$, then the appropriate finite elasticity BVP has no solution.

From the mathematical point of view the variational problem (1) is non-correct and, therefore, needs regularization. We use the partial regularization which is based on the special discontinuous finite-element approximation (FEA) [3]. After this discontinuous FEA the problem (1) is transformed into the non-linear system of algebraic equations which is badly determined. Therefore, for the numerical solution the decomposition method of adaptive block relaxation is used [2,4]. The main idea of this method consists of iterative improvement of zones with "proportional" fields by special decomposition of variables, and separate calculation on these variables.

The numerical results show that, for finding the main parameter t_* of a limited static load, the proposed technique has qualitative advantages over standard continuous finite-element approximations.

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L^2–Regularity Theory of Linear Strongly Elliptic Dirichlet Systems of Order $2m$ with Minimal Regularity in the Coefficients

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In this talk we consider the following Dirichlet system of order $2m$:

$$\begin{aligned} L(x,\nabla)u &= f(x) && \text{in } \Omega, \\ \nabla^k u &= 0 && \text{on } \partial\Omega \ (k = 0, \ldots, m-1). \end{aligned}$$

Here, Ω is a smooth bounded domain in $\mathbb{R}^n$ and the differential operator satisfies the Legendre–Hadamard condition. From the general elliptic theory we know that for sufficiently smooth coefficients and for $f \in H^{-m+s}(\Omega, \mathbb{R}^N)$, every weak solution $u \in H_0^m(\Omega, \mathbb{R}^N)$ is actually in $H^{m+s}(\Omega, \mathbb{R}^N)$ and satisfies an a–priori estimate of the following form:

$$\|u\|_{H^{m+s}(\Omega,\mathbb{R}^N)} \leq C_1 \|f\|_{H^{-m+s}(\Omega,\mathbb{R}^N)} + C_2 \|u\|_{L^2(\Omega,\mathbb{R}^N)}.$$

The latter a–priori estimate is of particular interest in applications to nonlinear PDE's (see e.g. [2] and [3]). There, the coefficients of $L(x,\nabla)$ result from a linearization procedure and consequently they cannot be chosen as smooth as one likes. Therefore, the authors cannot use the famous results stated in [1].

Here, we proove the above regularity result under minimal assumptions on the coefficients of $L(x,\nabla)$ and we give an explicit representation formula for the regularity constants C_1 and C_2.

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On the Solvability of Thermoviscoelastic Contact Problems with Coulomb Friction

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In a physically realistic model for contact problems with friction it is reasonable to include the aspect of heat transport, since friction usually generates a non–negligible amount of heat. In the present contribution we study a viscoelastic contact problem coupled with a heat conduction equation. The equations are coupled by volume terms describing the heat generated by viscous deformation and the deformation created by heat, and by a boundary term modeling the generation of heat by friction. In the contact problem the contact condition is formulated with the displacement velocities. The weak formulation of the problem consists of a variational inequality for the contact problem and a variational equation for the heat conduction problem.

The heat generated by friction can be modeled by a nonlinear non–monotone boundary term with quadratic growth in the variational formulation. If this model is used in a problem with linear constitutive laws for the material properties, then this boundary term dominates in estimates. This up–to–now prevents the proof of the existence of solutions for such a model. Here we consider two different simplifications:

1. In the first model we limit the growth of the heat generated by friction by a linear term and use linear constitutive laws for the viscoelastic material.

2. In the second model we admit the full quadratic growth of the heat generated by friction. In order to control this growth we employ a constitutive law for the viscoelastic material satisfying suitable growth conditions.

In both cases the problem is solved with a fixed point approach. Let $U(\vartheta)$ denote the solution of the contact problem with given temperature field ϑ and $\Theta(u)$ be the solution of the heat conduction problem with given displacement field u. If both problems are uniquely solvable, then the operator

$$\Phi := \Theta \circ U, \quad \Phi(\vartheta) = \Theta(U(\vartheta))$$

is well defined. Any solution of the coupled thermoviscoelastic contact problem is associated with a fixed point of this operator. However, the uniqueness of the solution to the contact problem is still an open problem. Therefore we employ the above described fixed point approach to an approximate problem obtained by the application of the penalty method. The existence of a fixed point for the approximate problems is proved with the fixed point theorem of Banach in the first case and the fixed point theorem of Schauder in the second case. The solvability of the original thermoviscoelastic contact problem follows by passing to the limit of the penalty parameter.

On the integrability of the Pfaffian equation concerning the octahedral web formed by four pencils of spheres

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In this paper, we consider four pencils of spheres whose centers lie on the same plane such that any three pencils cut the fourth pencil under a hexagonal 3-web [1,2]. We then use a system of Pfaffian equations to obtain the condition for the surface web formed by four pencils of spheres to be an octahedral web and show that this condition involves 28 parameters [1].

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Effect of large vibration amplitudes on the mode shapes and natural frequencies of thin circular cylindrical shells. A multi-mode approach

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A theoretical model has been proposed by Benamar et al. for studying the non-linear dynamic behaviour of thin straight structures like beams and plates [1,2,3,4]. The results obtained by these works has been in good agreement with previous theoretical and experimental investigations. The main topic of this paper is to present the mathematical development of a semi-analytical method, based on Hamilton's principle and spectral analysis, by extension of the previous model to the shell-type structures, in order to study the effects of large vibration amplitudes on the mode shapes and natural frequencies of thin circular cylindrical shells. The Donnell's non-linear theory is used, and, the longitudinal, circumferential and transverse displacement functions are expanded in the form of a series of basic functions satisfying the boundary conditions and harmonic motion is assumed. The non-linear dynamic variational problem obtained by applying Hamilton's principle is transformed into a static case by integrating the time functions over a period of vibration. The minimisation of the energy functional with respect to the basic function contribution coefficients has lead to a set of non-linear algebraic equations whose solution leads to the non-linear longitudinal, circumferential and transverse modes shapes, given as a function of the amplitudes of vibration, and their corresponding frequencies. Application of this approach was adapted, to the shells of infinite length, using a multi-mode approach, considering, first; transverse vibrations only [5], second;

coupling circumferential-transversal vibrations[6,7], and recently the simply supported circular cylindrical shells using a single mode approximation[8]. Because of their interest in many branch of engineering (aeronautical, ocean and civil), we will present same results, at multi-mode approach, for the simply supported circular cylindrical shells.

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Gain of Regularity for one nonlinear dispersive equation

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Using the technics of Craig - Kappeler - Strauss, cf.,[1] we study smoothness properties of solutions of some nonlinear equations of evolution dispersive.

We consider the nonlinear dispersive equation

$$\partial_t u + \partial_x f(u) = \epsilon \partial_x \beta(\partial_x u) - \delta \partial_x^3 u \tag{1}$$

with $x \in \mathrm{IR}$, $t \in [0, T]$ and T is an arbitrary positive time. The flux $f = f(u)$ and the (degenerate) viscosity $\beta = \beta(\lambda)$ are given smooth functions satisfying certain assumptions. The complete equation arises, at least for the diffusion $\beta(\lambda) = \lambda$ (the so called Korteweg - de Vries - Burgers equation)as a model of the nonlinear propagation of dispersive and dissipative waves in many different physical systems. We studied this case in [2–3]. In this work we obtained under certain additional condition on f and β a C^∞ solution $u(x, t)$ for $t > 0$ if the initial data $u(x, 0)$ decay faster than polinomially on $\mathrm{IR}^+ = \{x \in \mathrm{IR} : x > 0\}$ and have certain initial Sobolev regularity.

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Posters

Path Formulation for a Modal Family

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All persistent bifurcation diagrams in unfoldings of the modal family

$$g(x, \lambda) = \varepsilon x^4 + 2ax^2\lambda + \delta\lambda^2$$

are described using path formulation: each bifurcation problem in the unfoldings of g is reinterpreted as a λ-parametrized path in the universal unfolding of x^4. The space of unfolding parameters for the modal family is divided into regions where bifurcation problems are contact-equivalent and the bifurcation diagrams for these persistent problems are shown.

Bifurcation of Infinitely Many Fine Modes from Radially Symmetric Internal Layers.

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For a system of reaction-diffusion equations of activator-inhibitor type defined on an N-dimensional disk, it is shown that there exists a family of spherically symmetric stationary transition layer solutions when the activator diffuses very slowly compared with the inhibitor. Spectral analyses are also carried out for the linearization of the internal layers, which reveal that the layer solutions are unstable, and more importantly, that there exist infinitely many static bifurcations from the radially symmetric layer as the diffusion rate of the activator goes to zero. The wave length (l) of the bifurcated solutions becomes more fine as the diffusion rate (d) gets smaller. The relation between these two quantities is: $l \propto d^{1/4}$. This result proves the validity of a conjecture given in [1] for the special case where the domain is a disk.

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