Exercises

Partial Differential Equations

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Problem 21

Let $E(\lambda)$ be a spectral family. Prove that

(i) for any λ the operator $E(\lambda)$ is self adjoint;

(ii) for any fixed u the function $\lambda \longmapsto (E(\lambda)u, u)$ is nondecreasing.

Problem 22

Let $E(\lambda)$ be a spectral family. Prove that for any fixed u, v the function $\lambda \longmapsto (E(\lambda)u, v)$ is of bounded total variation. The total variation can be defined as

$$TV(f) \;=\; \sup\{\sum_{n=1}^{N} \|f(x_i) - f(x_{i-1})\| \mid x_0 < \ldots < x_N,\; N \in \mathbb{N}\}$$

(Estimate $((E(\lambda_1) - E(\lambda_2))u, v)$.)

Problem 23

Let A be a real, symmetric $(n \times n)$-matrix. What is the corresponding spectral family? Use this family to define e^{At}. Does this definition coincide with the usual one?

Problem 24

Prove with a Fourier ansatz a resolvent estimate for the operator

$$A \,:\, \mathcal{D}(A) \;=\; H^2(S^1) \;\subseteq\; L^2(S^1) \longrightarrow L^2(S^1), \qquad u \longmapsto u_{xx}$$

on the interval $(0, 2\pi)$ with periodic boundary conditions and conclude by theorem 3.6 that A generates a strongly continuous semigroup.