Exercises

Partial Differential Equations

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Problem 39

Consider the equation

$$u_t(t) = Au(t) + f(t)$$

on the Banach space X with $u(0) = u_0$, $A \in G(M, \beta)$, and continuous f. Assume $\beta < 0$ and the existence of the limit

$$f_\infty := \lim_{t \nearrow \infty} f(t).$$

Prove the existence on an asymptotic limit of the mild solution u:

$$u_\infty := \lim_{t \nearrow \infty} u(t).$$

Which equation is solved by u_∞?

Problem 40

Let $u^n(\cdot)$ be mild solutions of

$$u_t^n(t) = Au^n(t) + f^n(t)$$

with initial values $u^n(0) = u_0^n$. Assume the existence of the limits

$$\lim_{n \to \infty} u_0^n = u_0 \quad \text{in } X; \qquad \lim_{n \to \infty} f^n = f \quad \text{in } C^0([0, \tau], X).$$

Do the solutions u^n then converge to a limit solution u in $C^0([0, \tau], X)$? What can you say about $t \in [0, \infty)$?

Problem 41

Consider the equation

$$u_t(t) = Au(t) + f(u(t))$$

with $A \in G(M, \beta)$ on the Banach space X. Let f be locally Lipschitz with the linear growth condition

$$\|f(u)\|_X \leq C(1 + \|u\|_X)$$

Prove the existence of a global mild solution $u(t)$, $t \in [0, \infty)$. (In particular, there exists a global mild solution for nonlinearities f that are *globally* Lipschitz.) Find a condition on f that provides global *strong* solutions.

Problem 42

Consider the equation

$$u_t(t) = Au(t) + f(u(t))$$

and the shift operator

$$(S_\tau u)(t) = u(t + \tau)$$

Prove: if u is a mild solution on $t \in [0, T_1]$ and $S_{T_1} u$ is a mild solution on $t \in [0, T_2]$ then u is a mild solution on $t \in [0, T_1 + T_2]$, i.e. mild solutions can be concatenated.