Homework assignment

Dynamical Systems

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http://dynamics.mi.fu-berlin.de/lectures/

due date: Friday, May 7, 2010

Problem 9: Identify the ω-limits of each trajectory of

(i) $\dot{x} = \cos(x), \qquad x \in \mathbb{R},$

(ii) $\begin{pmatrix} \dot{x_1} \\ \dot{x_2} \end{pmatrix} = \begin{pmatrix} \sin x_2 \\ \sin x_1 \end{pmatrix}, \qquad x = (x_1, x_2) \in \mathbb{R}^2.$

Problem 10: Consider an equilibrium x_0 of a flow φ_t in $X = \mathbb{R}^n$ and neighborhoods $U \subset V$ of x_0 in X such that each forward trajectory $\gamma(x) := \{\varphi_t(x), t \geq 0\}$ of a point $x \in U$ stays in V and converges to x_0 as $t \to \infty$.

Prove or disprove: Every first integral of φ is constant in U.

Problem 11: A (point-sized) professor —trying to escape from his office hours— starts from his office at the origin $(x, y) = (0, 0)$ of the plane $\mathbb{R}^2$ and runs along the positive x-axis with speed 1. At the same moment a (point-sized) student —with loads of tricky questions concerning the homework assignment— starts at the point $(x, y) = (0, 2)$ and chases the professor. The student has the same speed 1 and always runs directly towards the professor.

How close does the student get? Will her/his questions ever be answered?

Hint: Use appropriate coordinates (e.g. r = distance of student and professor, φ = angle of the x-axis with the connecting line of both persons) and solve the resulting system by separation of variables.

Problem 12: Consider the two vector fields

(i) $\dot{x} \;=\; f(x) \;:=\; x \;\in\; \mathbb{R}^2,$

(ii) $\dot{\xi} \;=\; g(\xi) \;:=\; (1 + |\xi|^2)\xi \;\in\; \mathbb{R}^2.$

Prove or disprove that f-orbits and g-orbits coincide.

Hint: The theorem stated in the lecture is not applicable as the “Euler-multiplier” is not bounded.