Homework assignment

Dynamical Systems

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 13: Solve the following initial-value problems by separation of variables and determine the maximal time intervals of existence of the solutions:

(i) $\dot{x} = x^2 e^t, \qquad x(0) = 1,$

(ii) $\dot{x} = 1 + x^2, \qquad x(0) = 0,$

(iii) $\dot{x} = 4 - x^2, \qquad x(0) = 0,$

(iv) $\dot{x} = x(2-x)^2, \qquad x(0) = 1,$

Problem 14: Consider a radially symmetric vector field in the plane,

$$\begin{aligned} \dot{x} &= f(x^2+y^2)x - g(x^2+y^2)y, \\ \dot{y} &= g(x^2+y^2)x + f(x^2+y^2)y. \end{aligned}$$

(i) Find an Euler multiplier $\mu = \mu(x^2+y^2)$ that turns it into a divergence-free vector field.

(ii) Find an example of the above form that does *not* possess a nontrivial First Integral.

(iii) What is wrong?

Problem 15: Consider the closed, sealed-off Müggelsee with predator and prey fish of positive total masses x and y, respectively. Suppose their dynamics obeys the Volterra-Lotka equations

$$\begin{aligned} \dot{x} &= x(\mu - \nu y), \\ \dot{y} &= y(-\varrho + \sigma x), \end{aligned}$$

with positive fixed parameters $\mu, \nu, \varrho, \sigma$. Very ($\varepsilon$-)cautious fishing would change μ into $\tilde{\mu} = \mu - \varepsilon$ and ϱ into $\tilde{\varrho} = \varrho + \varepsilon$, with $\varepsilon > 0$. Why?

Does the time-averaged prey population

$$\overline{x} := \lim_{t\to\infty} \frac{1}{t} \int_0^t x(\tau)\, \mathrm{d}\tau$$

increase or decrease, due to fishing? What happens to the total population $\overline{x+y}$?

Hint: Consider time averages of $\dot{x}/x$, $\dot{y}/y$.

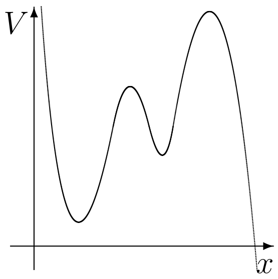
Problem 16: Sketch the phase portraits of $\ddot{x} + V'(x) = 0$,

(i) for the Kepler problem,

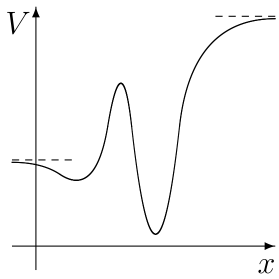
$$V(x) = -\frac{1}{x} + C\frac{1}{x^2},$$

$C > 0$, $x > 0$

(ii)



(iii)



Pay attention to saddle equilibria, homoclinic orbits, and asymptotic behavior at infinity.

Extra credit: explain why (i) is indeed the Kepler problem and describes the motion of a planet.