Homework assignment

Dynamical Systems

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Problem 17: [Duffing equation] Consider the potential

$$V(x) = x^4 - 2x^2 + 1,$$

and the associated pendulum equation

$$\ddot{x} = -V'(x)$$

Prove or dispove: For every $p > 0$ there exists a periodic orbit that is symmetric to the origin and has minimal period p.

Problem 18: Consider the pendulum equation

$$\ddot{x} + g(x) = 0$$

for a continuous, odd function $g : \mathbb{R} \to \mathbb{R}$, i.e. $g(-x) = -g(x)$ for all $x \in \mathbb{R}$. Assume $g(x) \cdot x > 0$ for all $x \neq 0$. Let $p(g, a) > 0$ be the minimal period of the solution to the initial value $x(0) = a > 0,\ \dot{x}(0) = 0$.

Prove:

(i) If $g_1(x) < g_2(x)$ for all $x > 0$ then $p(g_1, a) > p(g_2, a)$ for all $a > 0$.

(ii) [Hard spring] If $x \mapsto g(x)/x$ is strictly monotonically increasing for $x > 0$, then $a \mapsto p(g, a)$ is strictly monotonically decreasing for $a > 0$.

Hint: $y(t) := \frac{a_1}{a_2} x(t)$ solves the equation $\ddot{y} + \tilde{g}(y) = 0$ with $\tilde{g}(y) := \frac{a_1}{a_2} g(\frac{a_2}{a_1} y)$.

Problem 19: Use your favorite numerical integrator (e.g. `XPPAUT`) to plot the solution of the Van-der-Pol oscillator

$$\begin{aligned} \varepsilon^2 \dot{x} &= -y + x(1 - x^2) \\ \dot{y} &= x \end{aligned}$$

with initial conditions $x(0) = 1$, $y(0) = 0$ up to time $t = 10$. Choose parameters $\varepsilon = 0.4$, 0.1, 0.05, 0.03, and 0.025. Use stepsize $h = 10^{-3}$. Compare explicit Euler, Runge Kutta, and another solver of your choice. Describe and discuss observations and problems.

Extra credit: What happens for $\varepsilon = 0.03$ to the numerical solution calculated by the Runge-Kutta solver with fixed stepsize $h = 10^{-3}$?

Problem 20: Consider the synthesis of ammonia

$$N_2 + 3H_2 \leftrightarrows 2NH_3$$

Denote the reaction rates by $k_{\rightarrow}$, $k_{\leftarrow}$ and determine the kinetic equations for the vector $x = (x_{N_2}, x_{H_2}, x_{NH_3})$ of concentrations. Determine the ω-limit to initial data $x^0 = (x^0_{N_2}, x^0_{H_2}, 0)$.

Extra credit: Which choice of initial data yields the highest gain?