Homework assignment

Dynamical Systems

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 25: Consider a continuously differentiable vector field $f:\ X \times \mathbb{R} \to X = \mathbb{R}^n$. Let $x(t, t_0)$ denote the solution at time t of the associated initial-value problem

$$\dot{x}(t) = f(x(t), t), \qquad x(t_0) = x_0.$$

Prove: For any fixed τ such that $x(\tau, t_0)$ exists, there exists a neighborhood U of t_0 such that the map

$$(t_0 - \varepsilon, t_0 + \varepsilon) \to X, \qquad s \mapsto x(\tau, s) = \Phi_{\tau,s} x_0,$$

is differentiable with respect to s, for $s \in U$. Which differential equation is solved by $v(t) := \mathrm{D}_{t_0} x(t, t_0)$?

Problem 26: Consider the Banach space BC^1 of continuously differentiable vector fields $f:\ X \to X = \mathbb{R}^n$ with

$$\|f\|_{BC^1} \ :=\ \sup_{x \in X} (|f(x)| + |f'(x)|)\ <\ \infty.$$

Let f, g be vector fields in BC^1 and $x(f, t)$ denote the solution at time t of the differential equation

$$\dot{x}(t) = f(x(t)), \qquad x(0) = x_0.$$

Is the map

$$x(t, \cdot):\ BC^1 \to X, \qquad f \mapsto x(t, f),$$

differentiable with respect to $f \in BC^1$, for fixed t ? If so then which differential equation is solved by the variation $v(t) := \mathrm{D}_f x(t, f) g$?

Problem 27: Prove or disprove: The linearization of a flow, at an equilibrium $x_0 = 0$, is the flow of the linearized vector field, at $x_0 = 0$.

Problem 28: Let $A = (a_{ij})_{1 \le i,j \le n}$ be a real $(n \times n)$-matrix. Prove: The coefficients of the matrix e^{At} are non-negative for all $t \ge 0$ if, and only if, $a_{ij} \ge 0$ for all $i \ne j$.

Hint: It suffices to consider the case $a_{ij} \ge 0$ for *all* i, j. (Why?)