Homework assignment

Dynamical Systems

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Problem 45: Let f be a differentiable vector field such that each trajectory is bounded.

Prove or disprove: The ω-limit depends continuously on the initial condition, i.e. if

$$\lim_{n\to\infty} \operatorname{dist}(x_n, x) \;=\; 0,$$

then

$$\lim_{n\to\infty} \operatorname{dist}(\omega(x_n), \omega(x)) \;=\; 0.$$

Here we use the symmetric Hausdorff distance

$$\operatorname{dist}(A, B) \;:=\; \max\left(\sup_{a\in A} \inf_{b\in B} \operatorname{dist}(a,b)\,,\; \sup_{b\in B} \inf_{a\in A} \operatorname{dist}(a,b) \right).$$

Problem 46: Prove or disprove the theorem of Poincaré & Bendixson for flows on

(i) the sphere S^2,

(ii) the torus T^2.

Problem 47: [Arnol'd, (Russian) sample examination problems] To stop a boat at a dock, a rope is thrown from the boat which is then wound around a post attached to the dock. What is the breaking force on the boat if the rope makes 3 turns around the post, if the coefficient of friction of the rope around the post is 1/3, and if a dockworker pulls at the free end of the rope with a force of 100 N (approximately the force one needs to lift 10kg of potatoes)?

Problem 48: Consider the Van-der-Pol oscillator

$$\ddot{x} + \alpha(x^2 - 1)\dot{x} + x = 0$$

with positive parameter $\alpha > 0$. Prove the existence of a periodic orbit.

Hint: Apply the theorem of Poincaré & Bendixson. Use polar coordinates (r, φ) to discuss the dynamics near the origin and rescaled coordiantes (ϱ, φ), $\varrho = r^{-1}$, to discuss the dynamics far away from the origin.