Homework assignment

Dynamical Systems II

Bernold Fiedler, Stefan Liebscher

http://dynamics.mi.fu-berlin.de/lectures/

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Problem 1: [Floquet theory for discrete dynamical systems] Consider the iteration

$$x_{k+1} \;=\; A_k x_k$$

with $A_{k+p} = A_k$ for all $k \in \mathbb{N}$, and some fixed period $p \in \mathbb{N}$. Assume all matrices A_k to be invertible.

Prove: there exist matrices B, C_k, $k \in \mathbb{N}$ with $C_k = C_{k+p}$ and

$$\prod_{k=0}^{m} A_k = C_m B^m \qquad \text{for all} \quad m \in \mathbb{N}.$$

Compare with the Floquet theorem for flows and discuss.

Problem 2: (a) Consider the non-autonomous linear system $\dot{y}(t) = A(t)y(t)$, with periodic matrix A, $A(t+p) = A(t)$. For given initial time t_0, the Floquet theorem yields a solution $y(t) = W(t,t_0)y(t_0)$ of the form

$$W(t,t_0) \;=\; \mathrm{e}^{B_{t_0}t} Q_{t_0}(t)$$

with constant matrix B_{t_0} and periodic matrix Q_{t_0}, $Q_{t_0}(t+p) = Q_{t_0}(t)$. Prove that the matrix B_{t_0} is independent of the initial time t_0. How does Q_{t_0} depend on t_0 ?

(b) Consider the autonomous vector field $\dot{x} = f(x)$. Let S be a Poincaré section to a given periodic orbit γ of this system. Prove that the Floquet exponents of γ are independent of the choice of the Poincaré section S.

Problem 3: Let $I \subset \mathbb{R}$ be an interval and $A \in C^1(I, \mathbb{R}^{n\times n})$.

Prove: If A and $\dot{A}$ commute, i.e. if $[A(t), \dot{A}(t)] := A(t)\dot{A}(t) - \dot{A}(t)A(t) = 0$ for all $t \in I$, then

$$\frac{\mathrm{d}}{\mathrm{d}t} e^{A(t)} \;=\; \dot{A}(t)\, e^{A(t)} \;=\; e^{A(t)}\, \dot{A}(t).$$

Problem 4: Can an unstable equilibrium position become stable upon linearization? Can it become asymptotically stable? Can an asymptotically stable equilibrium become unstable?

Reminder: An equilibrium x is called stable under a flow Φ_t if for every $\varepsilon > 0$ there exists $\delta > 0$ such that forward orbit of the δ-neighborhood of x under Φ_t remains a subset of the ε-neighborhood of x. The equilibrium x is called asymptotically stable if in addition there exists a neighborhood of x such that $\{x\}$ is the ω-limit set of each point in that neighborhood.