Homework assignment

Dynamical Systems II

Bernold Fiedler, Stefan Liebscher

http://dynamics.mi.fu-berlin.de/lectures/

due date: Thursday, Nov 4, 2010

Problem 5: [Billard] An ideal point-sized billard ball moves with constant speed $v \in S^1$ on the unit disc $B^2 \subset \mathbb{R}^2$. The billard ball is reflected at the boundary $\partial B^2 = S^1$.

(i) For which initial conditions $(x_0, v_0) \in B^2 \times S^1$ is the set of reflections points of the billard trajectory dense on the boundary S^1 ?

(ii) Describe the ω-limit set in B^2 of each billard trajectory — either by numerical observation or by analytical insight.

Problem 6: Let $0 < \beta < 1$ be irrational and

$$s_n := \operatorname{sign}(\sin(n\pi\beta)), \qquad n = 1, 2, 3, ...$$

The sequence

$$w_n := |s_n - s_{n+1}|/2$$

detects the sign changes of the sequence s_n. Prove:

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} w_n = \beta.$$

Free extra: Is it possible to recover a rational number β from the sequence s_n?

Problem 7: Let $f : S^1 \to S^1$ be a homeomorphism of the circle that reverses orientation, i.e. the induced map $F : \mathbb{R} \to \mathbb{R}$ on the covering space $\mathbb{R}$ satisfies $F(x+2\pi) = F(x) - 2\pi$ for all $x \in \mathbb{R}$.

Prove or disprove: f has a fixed point.

Problem 8: In class "we" claimed that any flow φ_t on the 2-torus without equilibria possesses a global transverse section S, i.e. a closed curve that intersects every trajectory and is everywhere transverse to the vector field.

As a counterexample, find a flow φ_t on the 2-torus with (two or more) periodic orbits — but without equilibria — such that no global transverse section S exists.