

Homework assignment
Dynamical Systems II
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<http://dynamics.mi.fu-berlin.de/lectures/>
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Problem 9: Consider the vector field

$$\dot{y} = f(y), \quad y \in S^1, \quad f(y) > 0$$

with corresponding flow φ_t . Determine a formula for the rotation number $\varrho(\varphi_T)$ of the time- T -map. Does a devil's staircase arise?

Problem 10: Calculate the rotation number $\varrho(\alpha)$ of the time- 2π -map of the differential equation

$$\dot{x} = \alpha + \sin(x - t), \quad x \in S^1.$$

Problems 11 and 12: Let a, b denote C^2 -diffeomorphisms of S^1 and ϱ the rotation number. Prove or disprove:

- (i) $\varrho(a) + \varrho(a^{-1}) = 0$;
- (ii) $\varrho(aba^{-1}) + \varrho(b^{-1}) = 0$;
- (iii) if $\varrho(a) = \varrho(b) \notin \mathbb{Q}$ then there exists a homeomorphism h such that $b = hah^{-1}$;
- (iv) if $ab = ba$ and $\varrho(a) \notin \mathbb{Q}$ then $\varrho(ab) = \varrho(a) + \varrho(b)$;
- (v) if $\varrho(a) = \varrho(b) \notin \mathbb{Q}$ then $ab = ba$;
- (vi) if $\varrho(a) = \varrho(b) \notin \mathbb{Q}$ then $\varrho(ab) = 2\varrho(a)$;
- (vii) $\varrho(ab) = \varrho(ba)$;
- (viii) $\varrho(aba^{-1}b^{-1}) = 0$;
- (ix) if $n \in \mathbb{Z}$ then $\varrho(a^n) = n\varrho(a)$;
- (x) $\varrho(ab) = \varrho(a) + \varrho(b)$.