Homework assignment

Dynamical Systems II

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http://dynamics.mi.fu-berlin.de/lectures/

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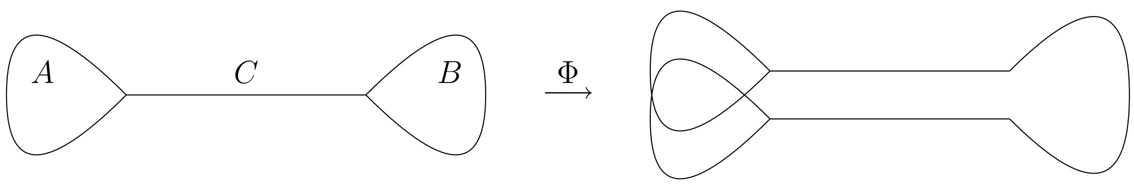
Problem 33: Let Φ be a diffeomorphism of the plane $\mathbb{R}^2$ with a transverse homoclinic orbit. In class, we found a shift on only two symbols for an iterate Φ^n. Prove that for every $m \in \mathbb{N}$ the shift of m symbols is conjugate to some iterate Φ^n on a suitable subset of $\mathbb{R}^2$.

Problem 34: Sketch the stable and unstable manifolds to an equlibrium with a transverse homoclinic point of a diffeomorphism of your choice. Extend the manifolds as far as you find interesting, but consistently with the λ-lemma.

Problem 35: Try to realize the following graph by a continuous map Φ (similar to the case of the Plykin attractor)

$$\begin{aligned} A &\to A \\ B &\to A \\ C &\to C + B - C \end{aligned}$$



Why does this construction not yield a hyperbolic attractor?

Problem 36: Consider the Plykin attractor defined in class as the ω-limit set of an initial domain $M = A \cup B \cup C \cup D$ under a diffeomorphism Φ,

$$P := \bigcap_{n=0}^{\infty} \Phi^n M,$$

The domain M is compact, connected, and path-connected. The same holds true for all iterates $\Phi^n M$. Prove or disprove:

(i) The Plykin attractor is connected.

(ii) The Plykin attractor is path-connected.

Reminder: A set X is called connected if it cannot be split into two nontrivial open sets, i.e. $U \subset \mathbb{R}^2$ is connected if for all open sets $U, V \subset \mathbb{R}^2$

$$\big(X \subset U \cup V \text{ and } U \cap V = \emptyset \implies X \subset U \text{ or } X \subset V.\big)$$

A set X is called path-connected if every two points in X can be connected by a continuous path, i.e.

$$\forall x, y \in X\ \exists f : [0,1] \to X \text{ continuous } \ : \ f(0) = x \text{ and } f(1) = y.$$