Homework assignment

Dynamical Systems II

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 37: Consider matrices $A \in SL(2,\mathbb{Z})$ with integer coefficients and determinant 1. These define linear orientation- and area-preserving maps on the plane.

(i) Prove that every $A \in SL(2,\mathbb{Z})$ defines a diffeomorphism of the torus $T^2 = \mathbb{R}^2/\mathbb{Z}^2$.

(ii) Is $A = \mathrm{id} \in SL(2,\mathbb{Z})$ structurally stable?

Extra credit: Which $A \in SL(2,\mathbb{Z})$ are structurally stable?

Problem 38: Consider the map

$$\Phi \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{2011} \begin{pmatrix} \sin(2\pi x)\cos(2\pi y) \\ \cos(2\pi x)\sin(2\pi y) \end{pmatrix}$$

on the torus $T^2 = \mathbb{R}^2/\mathbb{Z}^2$. How many homoclinic orbits does the point $(0,0) \in T^2$ have under iterations of Φ (zero, finitely many, countably many, more than countably many)?

Problem 39: Is the map

$$G : S^1 \to S^1 = \mathbb{R}/2\pi\mathbb{Z}, \qquad G(\varphi) = \varphi + \frac{1}{\pi^2}\cos^3\varphi,$$

C^1-structurally stable?

Problem 40: Consider a diffeomorphism Φ, a hyperbolic equilibrium x, and a homoclinic point p,

$$x \neq p \in W^s(x) \cap W^u(x).$$

Let $S(x,p)$ denote the segment of $W^s(x)$ connecting x and p. Let $U(x,p)$ denote the segment of $W^u(x)$ connecting x and p. We call p (and its orbit) *one-homoclinic*, if there are no additional intersections of these segments, i.e.

$$S(x,p) \cap U(x,p) = \{x,p\}.$$

How many one-homoclinic orbits does the Anosov map

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} : T^2 \to T^2.$$

possess?