Homework assignment

Dynamical Systems II

Bernold Fiedler, Stefan Liebscher

http://dynamics.mi.fu-berlin.de/lectures/

due date: Thursday, Jan 27, 2011

Problem 41: Consider a complex differentiable function $f : \mathbb{C} \to \mathbb{C}$, with $f(z_0) = 0$. Assume that f is not the constant function, $f \not\equiv 0$.

Prove that for small enough $\varepsilon > 0$ the degree $\deg(f, U_\varepsilon(z_0), 0)$ is given by the algebraic multiplicity of z_0 as a zero of f.

Problem 42: Consider a C^2 flow Φ_t on $\mathbb{R}^N$. Let the unit ball $B_1(0) \subset \mathbb{R}^N$ be positively invariant under Φ_t. Prove that Φ_t has an equilibrium in the closed unit ball.

Extra credit: Let Φ_t be a continuous flow (i.e. Φ_t is a continuous family of diffeomorphisms, $\Phi_0 = \text{id}$, $\Phi_{t+s} = \Phi_t \Phi_s$). Does the claim still hold?

Problem 43: Consider a non-autonomous, time-periodic vector field

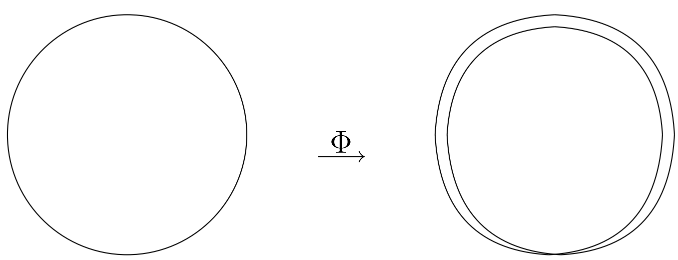
$$\dot{x} \;=\; f(t,x) \;\in\; \mathbb{R}^N, \qquad f(t+2\pi, x) = f(t,x),$$

with $x^{\mathrm{T}} f(t,x) \leq 0$ for all $|x|_2^2 = 10^6$ and $t \in \mathbb{R}$.

Prove that there exists a 2π-periodic orbit.

Problem 44: [Smale Solenoid] Realize the following graph by a continuous differentiable map Φ of the (filled) 2-Torus

$$A \;\to\; A + A$$



(i) Give an explicit expression of Φ in suitable coordinates.

(ii) Prove that the attractor is hyperbolic.