Homework assignment

Dynamical Systems II

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Problem 45: Let $\mathcal{M}^c = \operatorname{graph} \psi$ be a $\mathcal{C}^1$ center manifold of the flow Φ_t to the vector field

$$\dot{x} = Ax + g(x).$$

Here $\psi : E^c \to E^h$ where $\mathbb{R}^n = E^c \oplus E^h$ is the eigenspace decomposition w.r.t. A.

Prove that $\mathcal{M}^c$ is tangential to E^c, i.e. $\psi'(0) = 0$.

Hint: Use the invariance of $\mathcal{M}^c$ under the flow Φ_t.

Problem 46: Consider the vector field

$$\dot{x} \;=\; f(x) \;\in\; \mathbb{R}^n,$$

with equilibrium at the origin, $0 = f(0)$. Assume that the linearization $Df(0)$ possesses an algebraically simple eigenvalue 0 and all other eigenvalues have nonzero real part.

Prove or disprove that there exists a neighborhood U of the origin, such that U does not contain a non-stationary periodic orbit (i.e. every periodic solution in U is an equilibrium).

Problem 47: Consider the system of differential equations

$$\begin{aligned} \dot{x} &= xy, \\ \dot{y} &= -y + x^2. \end{aligned}$$

Use a (local) center manifold to decide whether the equilibrium $x = y = 0$ is asymptotically stable.

Note: Use the invariance of the center manifold to calculate the necessary terms of its Taylor expansion.

Problem 48: Consider the parameter-dependent vector field

$$\begin{aligned} \dot{x} &= 2(x+y)^2 + x - y - \lambda, \\ \dot{y} &= - x + y - \lambda. \end{aligned}$$

Calculate the local center manifold of the origin and the reduced vector field (that is their Taylor expansions of sufficient order) to determine the bifurcation diagram.