Homework assignment

Dynamical Systems III

Bernold Fiedler, Stefan Liebscher

http://dynamics.mi.fu-berlin.de/lectures/

due date: Friday, July 08, 2011, 13:00

(As usual, solve (at least) 2 problems, get 1 right.)

Problem 45: Consider the linear system

$$\dot{z} = A(t)z,$$

with $A(t) \to A_0$ for $t \to \pm\infty$. (Linearization along a homoclinic orbit yields such a system.) Let A_0 be hyperbolic. Prove that small bounded solutions $z(t)$ are in fact homoclinic solutions, i.e. $z(t) \to 0$ for $t \to \pm\infty$. (You may assume additional smoothness of $A(t)$.)

Problem 46: Consider Banach spaces X, Y. Prove that the set of compact linear operators $X \to Y$ is closed with respect to the operator norm.

Remark: The set of compact operators is in fact the closure of the set of operators with finite-dimensional range.

Problems 47 and 48: Consider the non-autonomous linear system

$$\dot{z} = A(t)z.$$

(i) Assume $A(t) = A_+$ for $t \geq 0$ and $A(t) = A_-$ for $t < 0$ with hyperbolic $A_\pm$.

(ii) Assume the convergence $\lim_{t\to\pm\infty} A(t) = A_\pm$ to hyperbolic $A_\pm$ with exponential rate.

Prove, in each case, that the associated operator

$$\mathcal{L} : BC^1 \to BC^0, \qquad \mathcal{L}z = \dot{z} - A(t)z$$

is a Fredholm operator. Prove that the Fredhom index is given by the difference of the dimensions of the unstable generalized eigenspaces of $A_\pm$.