Basic Questions of Dynamical Systems II

1. What is a Poincaré section to a periodic orbit of a flow?
2. What is a Poincaré map to a periodic orbit of a flow?
3. Formulate the Floquet theorem for a non-autonomous, time periodic, linear differential equation.
4. Formulate the Floquet theorem for an autonomous vector field, linearized at a periodic orbit.
5. What are Floquet multipliers and Floquet exponents of a periodic orbits of an autonomous vector field?
6. Why do periodic orbits of an autonomous vector field possess a trivial Floquet multiplier 1?
7. How is the rotation number of an (orientation preserving) homeomorphism $f : S^1 \to S^1$ defined?
8. How are existence and periods of periodic points related to the rotation number of a homeomorphism $f : S^1 \to S^1$?
9. Formulate the theorem of Denjoy for C^2-diffeomorphisms $f : S^1 \to S^1$.
10. How are local/global stable and unstable manifolds on a hyperbolic equilibrium of a vector field defined?
11. Formulate the theorem on the existence of local stable and unstable manifolds to a hyperbolic equilibrium of a vector field.
12. Formulate the theorem on the existence of local stable and unstable manifolds to a hyperbolic fixed point of a diffeomorphism.
13. Are stable and unstable manifolds to a hyperbolic equilibrium unique? What are the tangent spaces to stable and unstable manifolds at the equilibrium?
14. What is the (Bernoulli) shift on N symbols? Define the shift space, its topology, and the shift map.

15. How can we construct
    (a) periodic orbits of every period
    (b) a dense set of periodic orbits
    (c) a dense orbit

    for the shift on 2 symbols?

16. How does the shift on 2 symbols illustrate recurrence as well as sensitive dependence on initial conditions?

17. What is the Smale horseshoe?

18. Formulate the theorem on the embedding of the shift into a C^0 horseshoe.

19. Formulate the theorem on the embedding of the shift into a C^1 horseshoe.

20. Sketch a horseshoe construction for the bouncing-ball map

$$
\begin{aligned}
\Phi_{k+1} &= \Phi_k + v_k, \\
v_{k+1} &= v_k - \gamma \cos(\Phi_k + v_k),
\end{aligned}
$$

    under a suitable assumption on γ.

21. How is a hyperbolic structure defined?

22. What is a transverse homoclinic point of a diffeomorphism?

23. Formulate the λ-lemma.

24. How does a transverse homoclinic point give rise to shift dynamics? Sketch the relevant picture.

25. What is the Plykin attractor?

26. How is C^1 structural stability of a diffeomorphism defined?

27. Give at least two examples of structurally stable diffeomorphisms of the 2-torus.

28. Sketch the geodesic flow on the Lobachevsky plane. What are the horocycles? What is their dynamic significance?

29. What is a strange attractor? Sketch an example and list relevant properties.

30. Formulate Brouwer's fixed-point theorem.

31. How is the local center manifold to a non-hyperbolic equilibrium of a vector field defined?

32. Formulate the theorem on the existence of a local center manifold to a non-hyperbolic equilibrium of a vector field.

33. Formulate the theorem on the existence of a local center manifold to a non-hyperbolic fixed point of a diffeomorphism.

34. Under which assumptions on the vector field does a global center manifold to a non-hyperbolic equilibrium exist? Is the global center manifold unique?

35. Is the local center manifold to a non-hyperbolic equilibrium unique? What is the tangent space to a C^1 center manifold at the equilibrium?

36. How are the local center-stable and center-unstable manifolds to a non-hyperbolic equilibrium of a vector field defined? When do they exist?

37. Let A be the linearization of a C^1 vector field on $\mathbb{R}^n$ at the equilibrium $x = 0$. Suppose the only purely imaginary eigenvalue of A is a simple eigenvalue zero. Can the vector field possess periodic orbits arbitrarily near $x = 0$?

38. Let A be the linearization of a C^1 vector field on $\mathbb{R}^n$ at the equilibrium $x = 0$. Suppose the only purely imaginary eigenvalues of A are $\pm\mathrm{i}$, both simple. Can the vector field possess periodic orbits arbitrarily near $x = 0$?

39. How can the (global) center manifold be written as a fixed point of a contraction map on a suitable function space? Define the space and the contraction map.