Homework assignment

Infinite Dimensional Dynamical Systems

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http://dynamics.mi.fu-berlin.de/lectures/

due date: Tuesday, April 17, 2012

Problem 1: Study potentials and phase portraits of the autonomous Hamiltonian system

$$0 = v_{xx} + f(v),$$

for $f(v) = -a + v^2$ and all $a \in \mathbb{R}$.

Problem 2: Consider the pendulum equation

$$0 = v_{xx} + f(v),$$

for a continuous, odd function $f : \mathbb{R} \to \mathbb{R}$. Assume $f(v) \cdot v > 0$ for all $v \neq 0$. Let $T(f, a_+) > 0$ be the minimal period of the solution to the initial value $v(0) = a_+ > 0$, $\dot{v}(0) = 0$. Prove:

(i) If $f_1(v) < f_2(v)$ for all $v > 0$ then $T(f_1, a_+) > T(f_2, a_+)$ for all $a_+ > 0$.

(ii) [Hard spring] If $v \mapsto f(v)/v$ is strictly monotonically increasing for $v > 0$, then $a_+ \mapsto T(f, a_+)$ is strictly monotonically decreasing for $a_+ > 0$.

(iii) [Soft spring] If $v \mapsto f(v)/v$ is strictly monotonically decreasing for $v > 0$, then $a_+ \mapsto T(f, a_+)$ is strictly monotonically increasing for $a_+ > 0$.

Problem 3: Consider the equilibria of the PDE

$$u_t = u_{xx} + u(1 - u^2)$$

with Neumann boundary conditions on the interval $[0, L]$. Why are the bifurcation points from the trivial equilibrium $u \equiv 0$ at $L = \pi, 2\pi, \ldots$? Are the bifurcations sub- or supercritical, i.e. does L decrease or increase along the bifurcating, non-trivial branches?

Problem 4: Calculate the heteroclinic orbits of the Chafee-Infante ODE

$$0 = v_{xx} + v(1 - v^2).$$

How would you interpret these solutions for the PDE $u_t = u_{xx} + u(1 - u^2)$?