

Homework assignment
Infinite Dimensional Dynamical Systems

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<http://dynamics.mi.fu-berlin.de/lectures/>
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Problem 5: Determine the Sturm permutations σ of the Chaffee-Infante problem,

$$u_t = u_{xx} + u(1 - u^2), \quad 0 < x < L, \quad +\text{Neumann b.c.},$$

for all $L > 0$, $L \notin \pi\mathbb{Z}$.

Problem 6: Consider the problem

$$u_t = u_{xx} + f(x, u, u_x), \quad +\text{Neumann b.c.}$$

Show that the Sturm permutation σ_f on $0 < x < L$ coincides with σ_{f^L} on $0 < x < 1$, with

$$f^L(x, u, u_x) := L^2 f(Lx, u, L^{-1}u_x).$$

Problem 7: Consider the problem

$$u_t = u_{xx} + f(u), \quad +\text{Neumann b.c.}$$

Show that the Sturm permutation σ_f is an involution, i.e. $\sigma_f^2 = \text{id}$.

Problem 8: Consider the Chaffee -Infante problem,

$$u_t = u_{xx} + u(1 - u^2), \quad x \in [0, L], \quad u_x(0) = u_x(L) = 0.$$

Determine the Sturm-Liouville eigenvalues and the Morse indices of the trivial equilibrium $u \equiv 0$ and the homogeneous nontrivial equilibria $u \equiv \pm 1$. Compare PDE and ODE stability for each of the three equilibria.