Homework assignment

Infinite Dimensional Dynamical Systems

Bernold Fiedler, Stefan Liebscher
http://dynamics.mi.fu-berlin.de/lectures/

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Problem 13: Consider the problem

$$u_t \;=\; u_{xx} + f(x,u,u_x), \qquad \text{with Neumann boundary conditions.}$$

Sturm blocking 1 excludes a heteroclinic orbit $v_- \rightsquigarrow v_+$ between equilibria $v_\pm$, if there exists an equilibrium w, with $w(0)$ strictly between $v_-(0)$ and $v_+(0)$, such that $z(v_- - w) = z(v_+ - w)$.

Show that Sturm blocking 1, at $x = 0$, is equivalent to Sturm blocking 1, at $x = L$.

Problem 14: Consider the problem

$$u_t \;=\; u_{xx} + f(x,u,u_x), \qquad \text{with Neumann boundary conditions.}$$

Assume $v_\pm$ are hyperbolic equilibria, with $i(v_-) = i(v_+) + 1$. Assume they possess a heteroclinic orbit, $v_- \overset{u(t)}{\rightsquigarrow} v_+$. Show

$$z(u(t) - v_-) \;=\; z(u(t) - v_+) \;=\; z(v_+ - v_-) \;=\; i(v_+),$$

for all $t \in \mathbb{R}$.

Problem 15: Show that the pendulum flow

$$v_{xx} + f(v) \;=\; 0$$

for the Chafee-Infante nonlinearity $f(v) = v(1 - v^2)$ is not global. Can we still find all equilibria of

$$u_t \;=\; u_{xx} + f(u), \qquad \text{with Neumann boundary conditions,}$$

by the shooting curve γ, i.e. the image at $x = L$ in the (v, v_x) phase plane of the v-axis at $x = 0$ under the maximally defined local ODE flow?

Problem 16: Consider a compact, strongly continuous semiflow Φ^t on a Banach space X. The ω-limit set

$$\omega(u_0) \;=\; \bigcap_{t \geq 0} \operatorname{clos} \gamma^+(\Phi^t(u_0))$$

of a bounded forward trajectory $\gamma^+(u_0)$ is

(i) nonempty, (ii) compact, (iii) connected, (iv) invariant.

Show three of these four properties!