Homework assignment

Infinite Dimensional Dynamical Systems

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 17: Consider the problem

$$u_t \;=\; u_{xx} + f(x,u,u_x), \qquad \text{with Neumann boundary conditions.}$$

Assume hyperbolicity of all PDE equilibria, global ODE flow, and ODE dissipativity, i.e. $f(x,v,0)v < 0$ for large $|v|$.

Let γ be the shooting curve and σ the Sturm permutation. Prove that the index rule

$$i(v_{n+1}) \;=\; i(v_n) + (-1)^{n-1}\mathrm{sign}(\sigma^{-1}(n+1) - \sigma^{-1}(n))$$

means that right/left turns of γ increase/decrease the index by 1.

Problem 18: Consider the problem

$$u_t \;=\; u_{xx} + f(x,u,u_x), \qquad \text{with Neumann boundary conditions.}$$

Assume hyperbolicity of all PDE equilibria, global ODE flow, and ODE dissipativity, i.e. $f(x,v,0)v < 0$ for large $|v|$. Label equilibria $\mathcal{E} = \{v_1, \ldots, v_N\}$ such that $v_1(0) < \cdots < v_N(0)$.

Show that $i(v_n) \equiv n+1 \pmod 2$, i.e. the parities of $n+1$ and of the Morse index of v_n coincide.

Problem 19: Consider the problem

$$u_t \;=\; u_{xx} + f(x,u,u_x), \qquad \text{with Neumann boundary conditions.}$$

Assume hyperbolicity of all PDE equilibria, global ODE flow, and ODE dissipativity, i.e. $f(x,v,0)v < 0$ for large $|v|$. Label equilibria $\mathcal{E} = \{v_1, \ldots, v_N\}$ such that $v_1(0) < \cdots < v_N(0)$.

Show that $i(v_N) = 0$ and that the number $N = |\mathcal{E}|$ of equilibria is odd.

Problem 20: Consider the nonlinearity $f(u) = u(u+1)$, which violates our usual ODE dissipativity assumption. Starting from the ODE time map, study the shooting curve, for $0 < x < L$. Try to determine the PDE Morse indices $i(v_n)$ from our formula, starting from $v_1 \equiv -1$. Does the formula provide the correct Morse index $i(v_n)$ for $v_n \equiv 0$?