Homework assignment

Infinite Dimensional Dynamical Systems

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 21: Consider the problem

$$u_t \;=\; u_{xx} + f(x, u, u_x), \qquad \text{with Neumann boundary conditions.}$$

Assume hyperbolicity of all $N = 2m + 1$ PDE equilibria, global ODE flow, and ODE dissipativity, i.e. $f(x, v, 0)v < 0$ for large $|v|$.

(i) Show that $i(v_n) \le m$, for all $n = 1, \ldots, N$

(ii) Show that $\max_n i(v_n) = m$ if, and only if, the shooting permutation σ coincides with σ of the Chafee-Infante problem!

Problem 22: Consider the problem

$$u_t \;=\; u_{xx} + f(x, u, u_x), \qquad \text{with Neumann boundary conditions.}$$

Assume hyperbolicity of all PDE equilibria, global ODE flow, and ODE dissipativity, i.e. $f(x, v, 0)v < 0$ for large $|v|$.

Show that

$$\sigma \;=\; \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 2 & 5 & 4 & 3 & 6 & 7 \end{pmatrix} \;=\; (\,3\,5\,) \;\in\; S_7$$

is **not** a Sturm permutation.

Problem 23: [Strong monotonicity] Consider the problem

$$u_t \;=\; u_{xx} + f(x, u, u_x), \qquad f \in \mathrm{Sturm}^{\mathcal{N}}(x, u, u_x).$$

Let $u_1(t, x), u_2(t, x)$ be solutions with

$$u_1(0, x) \;>\; u_2(0, x) \qquad \text{for all } x.$$

Show that

$$u_1(t, x) \;>\; u_2(t, x) \qquad \text{for all } x \text{ and } t \ge 0.$$

Problem 24: Construct a dissipative meander permutation which is **not** Morse, and hence does **not** arise via ODE shooting.