Homework assignment

Infinite Dimensional Dynamical Systems

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http://dynamics.mi.fu-berlin.de/lectures/

due date: Tuesday, June 05, 2012

Problem 25: Are

$$\sigma_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 4 & 5 & 6 & 7 & 8 & 3 & 2 & 9 \end{pmatrix} = (2\,4\,6\,8)(3\,5\,7)$$

$$\sigma_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 6 & 7 & 8 & 5 & 2 & 3 & 4 & 9 \end{pmatrix} = (2\,6)(3\,7)(4\,8)$$

Sturm permutations? Determine the associated connection graphs $\mathcal{C}_1$ and $\mathcal{C}_2$. Are $\mathcal{C}_1$ and $\mathcal{C}_2$ isomorphic?

Problem 26: Let $v_\pm \in \mathcal{E}_f$ be equilibria of a Sturm PDE, $f \in \mathrm{Sturm}^{\mathcal{N}}(x, u, u_x)$, with adjacent boundary values at $x = 0$ or at $x = L$.

Show that $v_\pm$ possess a heteroclinic orbit connecting them.

Problem 27: In a Sturm PDE, $f \in \mathrm{Sturm}^{\mathcal{N}}(x, u, u_x)$, let $\underline{v}, \overline{v}$ denote the equilibria with lowest and highest boundary value at $x = 0$, respectively. Let w denote any other equilibrium. Prove or disprove,

$$\underline{v}(x) < w(x) < \overline{v}(x), \qquad \text{for all } x \in [0, L].$$

Extra credit: Solve this problem in at least two quite different ways.

Problem 28: Consider the permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 1 & 10 & 5 & 6 & 7 & 8 & 9 & 2 & 3 & 4 & 11 \end{pmatrix} = (2\,10\,4\,6\,8)(3\,5\,7\,9).$$

(i) Show that σ is Sturm.

(ii) Determine the connection graph $\mathcal{C}$ of σ.

(iii) Show that $\mathcal{C}$ is non-planar.