Homework assignment

Infinite Dimensional Dynamical Systems

Bernold Fiedler, Stefan Liebscher
http://dynamics.mi.fu-berlin.de/lectures/

Corrected versions of problems 30,31

Problem 29: Are

$$\sigma_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 4 & 5 & 8 & 7 & 6 & 3 & 2 & 9 \end{pmatrix} = (2\,4\,8)(3\,5\,7)$$
$$\sigma_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 6 & 7 & 8 & 5 & 4 & 3 & 2 & 9 \end{pmatrix} = (2\,6\,4\,8)(3\,7)$$

Sturm permutations? Determine the associated connection graphs $\mathcal{C}_1$ and $\mathcal{C}_2$. Are $\mathcal{C}_1$ and $\mathcal{C}_2$ isomorphic?

Problem 30: [Suspension I] For any permutation $\sigma \in S_N$ define the suspension $\hat{\sigma} \in S_{N+2}$ as

$$\hat{\sigma}(n) := \begin{cases} \sigma(N+2-n)+1, & \text{for } 2 \leq n \leq N+1, \\ n, & \text{for } n \in \{1, N+2\}. \end{cases}$$

(i) Show that $\hat{\sigma}$ is a dissipative meander, whenever σ is.

(ii) Show that the Morse numbers satisfy

$$\hat{i}_n = i_{n-1} + 1, \qquad \text{for } 2 \leq n \leq N+1.$$

In particular, any dissipative meander σ becomes Sturm after finitely many suspensions.

Problem 31: [Suspension II]

(i) Assume $\sigma \in S_N$ is a Sturm permutation with suspension $\hat{\sigma} \in S_{N+2}$. Show that the zero numbers satisfy

$$z(\hat{v}_n - \hat{v}_m) = \begin{cases} z(v_{n-1} - v_{m-1}) + 1, & \text{for all } 2 \leq m < n \leq N+1, \\ 0, & \text{for } 0 = m < n \text{ or } m < n = N+2. \end{cases}$$

Here $v_1, \ldots, v_N$ denote the equilibria of a Sturm attractor with shooting permutation σ, and $\hat{v}_1, \ldots, \hat{v}_{N+2}$ denote the equilibria of a Sturm attractor with shooting permutation $\hat{\sigma}$.

(ii) Describe the resulting connection graph $\hat{\mathcal{C}}$ of $\hat{\sigma}$, by comparision with the connection graph $\mathcal{C}$ of the Sturm permutation σ.

Problem 32: [Suspension III] Let $\hat{\sigma} \in S_{N+2}$ be a Sturm permutation, $N \geq 1$. Show that $\hat{\sigma}$ is the suspension of a Sturm permutation $\sigma \in S_N$ if, and only if, $\hat{\imath}_1$ and $\hat{\imath}_{N+2}$ are the only zero Morse numbers of $\hat{\sigma}$.