Problem 29:   Are

$$\sigma_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 4 & 5 & 8 & 7 & 6 & 3 & 2 & 9 \end{pmatrix} = (2\,4\,8)(3\,5\,7)$$

$$\sigma_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 6 & 7 & 8 & 5 & 4 & 3 & 2 & 9 \end{pmatrix} = (2\,6\,4\,8)(3\,7)$$

Sturm permutations? Determine the associated connection graphs $\mathcal{C}_1$ and $\mathcal{C}_2$. Are $\mathcal{C}_1$ and $\mathcal{C}_2$ isomorphic?

Problem 30:   [Suspension I] For any permutation $\sigma \in S_N$ define the suspension $\hat{\sigma} \in S_{N+2}$ as

$$\hat{\sigma}(n) := \begin{cases} N + 2 - \sigma(n-1), & \text{for } 2 \leq n \leq N+1, \\ n, & \text{for } n \in \{1, N+2\}. \end{cases}$$

(i) Show that $\hat{\sigma}$ is a dissipative meander, whenever σ is.

(ii) Show that the Morse numbers satisfy

$$i_n^{\hat{\sigma}} = i_{n-1}^{\sigma} + 1, \qquad \text{for } 2 \leq n \leq N+1.$$

In particular, any dissipative meander σ becomes Sturm after finitely many suspensions.

Problem 31:   [Suspension II]

(i) Assume $\sigma \in S_N$ is a Sturm permutation with suspension $\hat{\sigma} \in S_{N+2}$. Show that

$$z^{\hat{\sigma}}(v_n - v_m) = \begin{cases} z^{\sigma}(v_{N+2-n} - v_{N+2-m}) + 1, & \text{for all } 2 \leq m < n \leq N+1, \\ 0, & \text{for } 0 = m < n \text{ or } m < n = N+2. \end{cases}$$

(ii) Describe the resulting connection graph $\hat{\mathcal{C}}$ of $\hat{\sigma}$, by comparision with the connection graph $\mathcal{C}$ of the Sturm permutation σ.

Problem 32:   [Suspension III] Let $\hat{\sigma} \in S_{N+2}$ be a Sturm permutation, $N \geq 1$. Show that $\hat{\sigma}$ is the suspension of a Sturm permutation $\sigma \in S_N$ if, and only if, $\hat{i}_1$ and $\hat{i}_{N+2}$ are the only zero Morse numbers of $\hat{\sigma}$.