Homework assignment

Infinite Dimensional Dynamical Systems

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http://dynamics.mi.fu-berlin.de/lectures/

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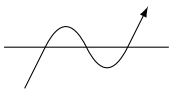
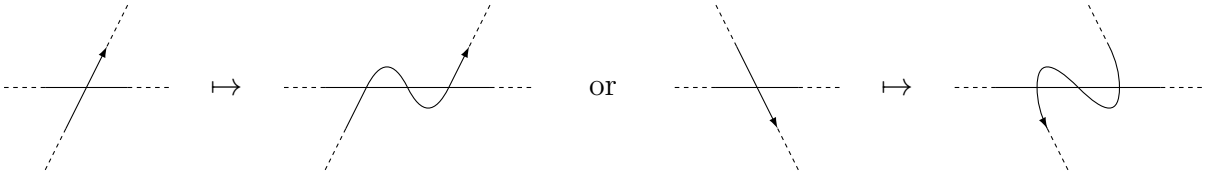
Problem 33: [Pitchforkability] A Sturm permutation $\sigma \in S_N$ is called pitchforkable if it arises from the trivial Sturm permutation $\varrho = \mathrm{id} \in S_3$ with meander curve



by a finite sequence of substitutions of ϱ:



(i) Show that the Chafee-Infante Sturm permutations $\sigma_{CI} \in S_N$, given by

$$\sigma_{CI}(n) := \begin{cases} n, & \text{for odd } n, \\ N+1-n, & \text{for even } n, \end{cases}$$

are pitchforkable, for any odd $N > 0$.

(ii) Show that the permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 1 & 8 & 7 & 2 & 3 & 6 & 9 & 10 & 5 & 4 & 11 \end{pmatrix} = (2\,8\,10\,4)(3\,7\,9\,5)$$

is Sturm, but not pitchforkable.

Extra credit: sketch the connection graph of σ.

Problem 34: Determine and draw a meander and the connection graph $\mathcal{C}$ of the Sturm permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 4 & 5 & 6 & 3 & 2 & 7 \end{pmatrix} = (2\,4\,6)(3\,5).$$

Did we overlook this permutation or this connection graph in our list of Sturm permutations $\sigma \in S_7$?

Problem 35: Show that the square is the 1-skeleton of the planar connection graph of only two Sturm permutations which are not related by trivial symmetries. Determine these Sturm permutations.

Problem 36: Consider the triangle as the 1-skeleton of a planar connection graph $\mathcal{C}$.

(i) Determine an associated Sturm permutation σ.

(ii) Determine the suspension $\hat{\sigma}$, see problem 30, and its connection graph $\hat{\mathcal{C}}$.

(iii) Show that $\hat{\mathcal{C}}$ is not a planar graph.

Extra credit: Show that $\hat{\mathcal{C}}$ is the only non-planar Sturm connection graph of a Sturm attractor with less than 11 equilibria.