Theoretical questions for the test on Differential Equations I.
Summer semester 2012

Basic notions

1. Basic notions of ODE: definitions of ODE (scalar and systems), solutions of ODE.
2. Examples of ODE and their solutions (phase spaces):
   a. population model,
   b. predator-prey model,
   c. pendulum.
3. Elementary methods of integration:
   a. separable equations,
   b. equations of differentials,
   c. linear first-order ODE.
4. Change of variables in ODE.
5. Equivalence of higher-order ODE and systems of first-order ODE.
6. ODE for complex-valued functions. Reduction to ODE for real-valued functions.

Linear theory

7. Linear first-order *homogeneous* systems with *constant* coefficients:
   a. theorem on general solution via Jordan chains,
   b. reduction to a systems with Jordan matrix by change of variables,
   c. exponents of matrices and exponential representation of solutions.
8. Linear *homogeneous* higher-order ODE with *constant* coefficients:
   a. main properties,
   b. theorem on general solution.
9. Linear first-order *homogeneous* systems with *variable* coefficients:
   a. theorem on existence of *n* linearly independent solutions,
   b. connection between two fundamental matrices for the same system,
   c. change of variables and connection between the fundamental matrices,
   d. Wronskian and Liouville's formula,
   e. application of Liouville's formula - the "volume formula".
10. Linear first-order *nonhomogeneous* systems with *variable* coefficients:
    a. variation-of-constants formula,
    b. particular case of constant coefficients.
11. Linear first-order *homogeneous* systems with *periodic* coefficients:
    a. main notions: equivalent systems, monodromy matrix,
    b. connection between two monodromy matrices for the same system,
    c. the monodromy matrix and equivalence of systems,
    d. Floquet-Lyapunov theorem on equivalence to a system with constant coefficients,
    e. Floquet multipliers and exponents, Lyapunov exponents,
    f. theorems on stability and instability of the zero solution.

Nonlinear theory

12. Local existence (and uniqueness) of solutions for nonlinear systems of ODE:
    a. Peano theorem: local existence (*continuous* right-hand side),
    b. Picard-Lindelöf theorem: local existence and uniqueness (*Lipschitz* right-hand side).
13. Global existence and uniqueness for *linear* systems.
14. Integral inequalities:
    a. proof by iterations
    b. Gronwall's lemma (without proof) and its application to uniqueness.
15. Extension of solutions, nonextendable solutions:
    a. theorem on existence and uniqueness of nonextendable solutions.
    b. theorem on leaving any compact.
    c. corollaries for solutions of *autonomous* systems with *finite* interval of existence:
        i. leaving any compact *in phase space*,
        ii. blow-up (in finite time).
16. Continuous dependence of solutions on parameters:
    a. w.r.t. parameters in the right-hand side,
    b. w.r.t. initial data.
17. Differentiability of solutions w.r.t. parameters:
    a. w.r.t. parameters in the right-hand side, equation of variations,
    b. w.r.t. initial data, equation of variations.

Autonomous (dynamical) systems

18. Basic properties of dynamical systems:
    a. time shift of solutions and the phase space: trajectories, vector fields,
    b. nonintersection of different trajectories,
    c. self-intersection of a trajectory implies equilibrium or cycle,
    d. criterion for $x(t)=a$ to be an equilibrium,
    e. flow.
19. Example: volume preserving systems with divergence-free vector fields.
20. Local qualitative behavior of a flow:
    a. definition of flow-equivalence,
    b. flow-box theorem,
    c. Grobman-Hartman theorem (without proof).
21. Classification of equilibria for linear systems on plane.
22. Lyapunov stability of equilibria:
    a. definition of stability and asymptotic stability,
    b. derivative along trajectory,
    c. Lyapunov function and a sufficient condition for (asymptotic) stability of equilibria,
    d. examples of systems admitting Lyapunov function.
23. Lyapunov theorem on stability of equilibria.
24. Lyapunov stability of periodic solutions:
    a. nonautonomous systems,
    b. autonomous systems.