Correction to homework assignment

Differentialgleichungen I - Problem Sheet 7

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http://dynamics.mi.fu-berlin.de/lectures/12SS-Gurevich-Dynamics/

Problem 25 (method of undetermined coefficients): In order to solve nonhomogeneous linear equations, we considered in class the general method of variation of constants. For certain right-hand side a simpler method, *the method of undetermined coefficients*, can be used.

(i) **(10 points)** Consider the equation

$$L(D)z = F(t), \tag{1}$$

where L is an n-order polynomial, and $F(t)$ is a continuous function for all $t \in \mathbb{R}$. Let z_p be a particular solution of (1), and $u(t)$ be a general solution of the homogenous equation $L(D)u = 0$. Show that $z = z_p + u$ is a general solution of (1).

(ii) **(90 points)** If the right-hand side of equation (1) has the form

$$P_m(t)e^{\gamma t},$$

where P_m is a polynomial of degree m, then a particular solution can be found in the form

$$x(t) = t^s Q_m(t)e^{\gamma t},$$

where Q_m is a polynomial of degree m, and s is equal to the multiplicity of the root γ of the characteristic polynomial of the homogeneous equation (if γ is not a root then $s = 0$). If the right-hand side of the form

$$e^{\gamma t}(P_m(t)\cos(\beta t) + Q_m(t)\sin(\beta t)),$$

then a particular solution can be found in the form

$$x(t) = t^s e^{\gamma t}(R_m(t)\cos(\beta t) + T_m(t)\sin(\beta t)),$$

where R_m, T_m are polynomials of degree m, and s is equal to the multiplicity of the root $\gamma + i\beta$ of the characteristic polynomial of the homogeneous equation (if $\gamma + i\beta$ is not a root then $s = 0$)

Task: In each of the following problems find the general solution.

(a) $y'' - 9y = e^{3x}\cos x$,

(b) $y'' - 5y' = 3x^2 + \sin 5x$,

(c) $x^2y'' - 2y = \sin \ln x$.