Homework assignment

Differentialgleichungen I - Problem Sheet 8

Pavel Gurevich, Eyal Ron, Sergey Tikhomirov

http://dynamics.mi.fu-berlin.de/lectures/12SS-Gurevich-Dynamics/

due date: Wednesday, June 06, 2012, at 13:00.

Announcement: Unfortunately a misprint has fallen in the theoretical background of question 25. A corrected version can be found on the website. As a result this question is postponed by a week. Please hand it in together with this exercise sheet.

Problem 28: Consider the following differential equation

$$\dot{x} = \begin{pmatrix} a & -b \\ b & a \end{pmatrix} x,$$

with $a, b \in \mathbb{R}$. Transform this linear differential equation into polar coordinates:

$$x = \begin{pmatrix} rcos\phi \\ rsin\phi \end{pmatrix},$$

with $r > 0$, $\phi \in \mathbb{R}/2\pi\mathbb{Z}$.

(i) Find necessary and sufficient conditions on a and b such that the solution $x = 0$ is stable/asymptotically stable.

(ii) Choose $b \neq 0$ arbitrary. Note that there are three types of behaviour depending on the value of a. Sketch the phase portraits in (x_1, x_2)-coordinates for each of the different types of behaviour.

Problem 29: Consider the differential equation

$$\dot{x} = A(t)x,$$

where $A(t+T) = A(t)$, $x \in \mathbb{R}^n$. For each of the following cases prove that the only bounded solution for $t \in \mathbb{R}$ is zero.

(i) All Lyapunov exponents are positive.

(ii) All Lyapunov exponents are nonzero.