Homework assignment

Differentialgleichungen I - Problem Sheet 11

Pavel Gurevich, Eyal Ron, Sergey Tikhomirov

http://dynamics.mi.fu-berlin.de/lectures/12SS-Gurevich-Dynamics/

due date: Wednesday, June 27, 2012, at 13:00.

Problem 37:

(i) Consider the differential equations

$$\dot{x} = f(x),$$

$$\dot{y} = f(y)g(t),$$

where $x, y \in R^n$, $f : R^n \to R^n$, $g : R \to R$, $f, g \in C^1$ and there exists $\epsilon > 0$ such that

$$\epsilon < g(t) < 1/\epsilon, \quad t \in R.$$

Let $x_0 \in \mathbb{R}^n$ be an initial data, and assume that $x(t)$ and $y(t)$ are the solutions of initial value problem $x(0) = x_0$ and $y(0) = x_0$ correspondingly. Consider the sets $A = \{x(t),\ t \in \text{interval of existence}\}$ and $B = \{y(t),\ t \in \text{interval of existence}\}$. Prove that the equality $A = B$ holds for any x_0.

(ii) Prove or disprove a similar statement for the differential equations

$$\dot{x} = x, \quad x \in R^2.$$

$$\dot{y} = (1 + |y|^2)y, \quad y \in R^2.$$

Problem 38: Consider the equation

$$\dot{x} = f(t, x),$$

where $f \in C^1(D)$, and $D \subset \mathbb{R} \times \mathbb{R}$ is an open set. Prove that for any compact set $K \subset D$ there exists an $h > 0$, such that for all $(t_0, x_0) \in K$, there is a unique solution starting at this point that exists for $t \in (t_0 - h, t_0 + h)$.

Hint: Note the connection between the change of initial data and the change of interval of existence.

Problem 39: Let $x \in R$ and consider the differential equation

$$\dot{x} = f(t, x),$$

where $f \in C^1$. Let $x(t, t_0, x_0)$ is the solution of the above differential equation with initial data $x(t_0) = x_0$. Prove or disprove that $\frac{\partial x}{\partial x_0}(t, t_0, x_0) \geq 0$ for any $t, t_0, x_0 \in R$.

Problem 40:

(i) Find $\frac{\partial y(x,\mu)}{\partial \mu}$, where $y(x,\mu)$ is a solution of the equation

$$y' = \mu x + \sin y$$

with the initial data $y(0,\mu) = 2\mu$ $(x, y, \mu \in R)$.

(ii) Consider the differential equation

$$y' = y^3 \sin x + y \cos x$$

with the initial data $y(0) = y_0 \in \mathbb{R}$.

Assume that $y_0 = 0$ and write an equation of variations for this equation and its initial data. Find the derivative of a solution with respect to the initial data y_0.