

Homework assignment

## Differentialgleichungen I - Problem Sheet 12

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<http://dynamics.mi.fu-berlin.de/lectures/12SS-Gurevich-Dynamics/>

due date: **Wednesday, July 4, 2012, at 13:00.**

**Problem 41:** Construct a function  $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ ,  $f \in C^1$ , such that the following conditions hold:

- $f(0) = 0$ ,
- $Df(0) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ,
- The trajectory of any non-zero solution of the differential equation

$$\dot{x} = f(x)$$

intersects with every arbitrary line that crosses the point 0 infinitely many times (roughly speaking, the trajectory looks like a spiral.)

**Bonus:** Is it possible to construct a map  $f$  of class  $C^2$ ?

**Problem 42:** Prove that the differential equations

$$\dot{x} = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} x \quad \text{and} \quad \dot{y} = \begin{pmatrix} a & -b \\ b & a \end{pmatrix} y,$$

where  $x, y \in \mathbb{R}^2$ ,  $a, b > 0$ , are  $C^0$ -flow equivalent.

**Problem 43:** Prove or disprove that flows generated by the differential equations

$$\dot{x} = 2x, \quad \dot{y} = 3y$$

- are  $C^0$ -flow equivalent.
- are not  $C^1$ -flow equivalent.