

Homework assignment

Differentialgleichungen I - Additional Questions

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<http://dynamics.mi.fu-berlin.de/lectures/12SS-Gurevich-Dynamics/>

due date: Friday, July 6, 2012, at the tutorial.

This problem sheet is for students who didn't reach the required 60% correct homework problem's solutions. Problems 1-4 are a chance to improve grades of problems from previous homework sheets. Problem 5 is a bonus question. It is also an excellent preparation for the exam.

Problem 1: (Replaces Problem 5) Find all the solutions of $\dot{x} = x^{1/3}$, $x(t_0) = 0$, $t_0 > 0$ for $t \geq 0$. Note that this equation does not have the property of uniqueness of solution.

Problem 2: (Replaces Problem 12) For each of the following equations find a general solution and all of their critical solutions. See theoretical background in Problem 12.

(i) $\dot{x}(t - \ln \dot{x}) = 1$,

(ii) $(\dot{x} + 1)^3 = (\dot{x} - x)^2$.

Problem 3: (Replaces Problem 18)

(i) Find a 2×2 matrix $A(t)$ that is continuously depending on t , and values $t_0 \in \mathbb{R}$ and $x_0 \in \mathbb{R}^2$, such that the function

$$x(t) = e^{\int_{t_0}^t A(t)} x_0 \tag{1}$$

is not a solution of the differential equation

$$x' = A(t)x. \tag{2}$$

(ii) **Bonus.** Assume additionally that for every $t_1, t_2 \in \mathbb{R}$ the equality $A(t_1)A(t_2) = A(t_2)A(t_1)$ holds. Prove that the function (1) is a solution of the differential equation (2).

Problem 4: (Replaces Problem 26) Find the bounded solution of the differential equation

$$x' = x + 1 + \sin t.$$

Problem 5: (Bonus 300 points) In each of the following equations find the general solution.

(i) $2xy' + y^2 = 1$.

(ii) $x^2y^3 + y + (x^3y^2 - x)y' = 0$.

(iii) $y' - y = 2x - 3$.

(iv) $y' = y^2 - 2/x^2$.

(v) $y'' - 5y' + 4y = 4x^2e^{2x}$.