

An example of the test on Differential Equations I

1. Linear homogeneous systems of first-order ODE with constant coefficients:

- (a) Write down a system and define its solution and its general solution.
- (b) Formulate and prove the theorem on the form of the general solution.

2. Formulate the Peano theorem on existence of solutions for the problem

$$\begin{cases} \dot{x} = f(t, x), \\ x(t_0) = x_0, \end{cases}$$

where $x(t) \in \mathbb{R}^n$.

3. Define (isochron) C^k -flow equivalence and formulate the Grobman–Hartman theorem.

4. Find the general solution of the equation

$$x'' - 9x = e^{3t} \cos t.$$

5. Find a solution of the Cauchy problem

$$\begin{cases} \dot{x} = \frac{1}{2t + 3x}, \\ x(0) = 1 \end{cases}$$

and indicate its maximal interval of existence.