Examination
on Differential Equations II

1. (<u>35 points</u>) Neumann problem for the Poisson equation:

   (a) Give a definition of a generalized solution.

   (b) Formulate and prove the theorem about the Fredholm solvability.

2. (<u>25 points</u>) Heat equation:

   (a) Give a definition of the anisotropic Sobolev space $H^{1,0}(Q_T)$.

   (b) Formulate the initial boundary-value problem (with the Dirichlet boundary condition) for the heat equation. Give a definition of a generalized solution.

   (c) Formulate the Cauchy problem for the heat equation. Give a definition of a classical solution in a strip
   $$\{(x,t) : x \in \mathbb{R}^n,\ 0 < t < T\}, \quad T > 0.$$

   (d) Formulate the theorem about the integral representation via the fundamental solution (Poisson formula).

   (e) Formulate and prove the theorem about the uniqueness of a bounded classical solution in a strip.

3. (<u>20 points</u>) Consider the differential equation
   $$uu_x + u_y = 1$$
   with the initial condition defined on the line $y = x$
   $$u(x,x) = x/2,$$
   where $u : \mathbb{R}^2 \to \mathbb{R}$. Find all characteristic points on the line $y = x$ and solve the equation in a neighborhood of noncharacteristic points.

4. (<u>20 points</u>) Solve the following initial boundary-value problem:
   $$\begin{cases} u_t = u_{xx} + 1 + \cos(3x), \quad x \in (0,\pi), \quad t > 0, \\ u_x|_{x=0} = u_x|_{x=\pi} = 0, \\ u|_{t=0} = 2. \end{cases}$$