

Homework assignment

**Differentialgleichungen II**

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<http://dynamics.mi.fu-berlin.de/lectures/12WS-Gurevich-PDE/>

**due date: 14:00, Wednesday, October 31, 2012**

**Problem 4:** Solve the following equation using characteristics

$$uu_{x_1} + u_{x_2} = 1, \quad u(x_1, x_1) = x_1/2.$$

**Problem 5:** In which region is it possible to solve the following equation using the method of characteristics? Find the solution in this region.

$$u_{x_1}^2 + u_{x_2}^2 = 4u, \quad u(0, x_2) = x_2^2.$$

**Problem 6:** Consider the equation

$$\frac{\partial}{\partial t}u + \frac{\partial}{\partial x}\left(\frac{u^2}{2}\right) = 0, \quad t > 0, x \in R \quad (1)$$

with the boundary condition

$$u(x, 0) = g(x), \quad x \in R, \quad (2)$$

where

$$g(x) = \begin{cases} 1 & \text{if } x \leq 0, \\ 1 - x & \text{if } x \in [0, 1], \\ 0 & \text{if } x \geq 1. \end{cases}$$

Find characteristics of this equation. Observe that two different characteristics may intersect.

(i) Find all points which lie on at least two characteristics.

(ii) Consider  $Q = \{(x, t), x \in R, t \in (0, 0.1)\}$ . Solve problem (1), (2) in  $Q$ .

**Bonus (100 points):** Consider the curve  $\gamma$  defined by  $(s(t), t)$  where

$$s(t) = (1 + t)/2, \quad t \geq 0.$$

Consider  $\Omega = \{(x, t) : x \in R, t \geq 0\} \setminus \gamma$ . Does there exist a function  $u \in C^1(\Omega)$  satisfying (1), (2) for  $(x, t) \in \Omega$ ?