Homework assignment
Differentialgleichungen II
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http://dynamics.mi.fu-berlin.de/lectures/12WS-Gurevich-PDE/
due date: 14:00, Wednesday, November 21, 2012

Problem 15:

(i) For which $\alpha > 0$ does the function $f(x) = |x|^{\alpha}$ belong to $L^2(Q)$, where

    (a) $Q = \{x \in R^n : |x| < 1\}$,

    (b) $Q = \{x \in R^n : |x| > 1\}$.

(ii) Does the function $f(x) = \frac{\sin x}{x^{5/4}}$ belong to $L^2(0,1)$?

(iii) For which α is the function $f(x) = \frac{x_1 x_2 \ldots x_n}{|x|^{\alpha}}$ Lebesgue integrable over the domain $Q = \{x \in R^n : |x| < 1\}$?

    Hint: A function $f(x)$ is Lebesgue integrable if and only if $|f(x)|$ is Lebesgue integrable.

Problem 16: Find all the eigenvalues and all the corresponding eigenfunctions of the following spectral problem

$$-e''(x) = \lambda e(x), \quad x \in (0,l), \tag{1}$$

$$e'(0) = 0, \quad e(l) = 0,$$

where $l > 0$.

Hint: Use the general form of the solution for equation (1).

Problem 17: Find the solutions of the following initial boundary-value problems

(i) $u_{tt} = u_{xx} + 2, \quad x \in (0,l), \quad u|_{x=0} = 0, \, u|_{x=l} = 0, \quad u|_{t=0} = u_t|_{t=0} = 0;$

(ii) $u_{tt} + 2u_t = u_{xx} - u, \quad x \in (0,\pi), \quad u_x|_{x=0} = u|_{x=\pi} = 0, \quad u|_{t=0} = 0, \, u_t|_{t=0} = x.$

Problem 18: Find the solutions of the following initial boundary-value problems

(i) $u_t = u_{xx}, \quad x \in (0,1), \quad u_x|_{x=0} = u_x|_{x=1} = 0, \, u|_{t=0} = x^2 - 1;$

(ii) $u_t = u_{xx} + u + 2\sin(2x)\sin x, \quad x \in (0,\pi/2), \quad u|_{x=0} = u|_{x=\pi/2} = 0, \, u|_{t=0} = 0.$