Homework assignment

# Differentialgleichungen III

# Problem Sheet 11

Pavel Gurevich, Eyal Ron

http://dynamics.mi.fu-berlin.de/lectures/13SS-Gurevich-Dynamics/

**Tutorial discussion date: Friday, July 5, 2013, at 10:00am**

**Problem 1:** Complete the details in the proof of the stable manifold theorem from the last lecture. In particular,

(i) Let

$$Pz(t) = e^{-L_2 t}a + \int_0^t e^{-L_2(t-s)}E_2 g(z(s))ds - \int_t^\infty e^{-L_1(t-s)}E_1 g(z(s))ds.$$

Show that $P$ is a contraction on $\Sigma_\rho$.

(ii) Prove that $\forall z_0 \in S$, $||z(t; z_0)||_\alpha \to 0$ as $t \to \infty$.

**Problem 2:** Consider the problem

$$\begin{cases} u_t = u_{xx} + \mu(u - au^3), & x \in (0, \pi),\ t > 0, \\ u|_{x=0} = u|_{x=\pi} = 0, \\ u|_{t=0} = u_0(x), \end{cases}$$

where $\mu$ is a nonnegative constant.

(i) Show that the equation defines a dynamical system on $\mathring{H}^1(0, \pi)$. **Hint:** Use an appropriate Liapunov function.

(ii) Assume that the set of equilibria for the problem is finite. Show that every solution goes to one of the equilibria as time goes to $+\infty$.

(iii) Show that for $\mu \in [0, 1]$ the origin is globally asymptotically stable in $\mathring{H}^1(0, \pi)$.