Homework Assignments

Dynamical Systems I

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 9: Suppose that a system on $X = \mathbb{R}^n$ has a periodic orbit Γ and an open neighborhood $\Gamma \subset U \subset X$ such that every solution $\Phi(\cdot, x_0)$, $x_0 \in U$ converges to Γ in forward time.

Prove or disprove: Every continuous first integral must be constant on U.

Problem 10: Consider the differential equation $\dot{x} = f(x)$, $f \in C^1(\mathbb{R}^n, \mathbb{R}^n)$. Consider the following modified differential equation

$$\dot{x} = \tilde{f}(x) = \frac{1}{1 + |f(x)|^2} f(x),$$

and its associated flow $\Phi : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$. Show that this flow for $\dot{x} = \tilde{f}(x)$ is defined for all $t \in \mathbb{R}$ (i.e. has unbounded times of existence). Show that the flow for $\dot{x} = f(x)$ has the same trajectories as the original system $\dot{x} = f(x)$.

Problem 11: Consider the differential equation

$$\dot{x} = A(x, y) \qquad \dot{y} = B(x, y),$$

where $A, B \in C^1(\mathbb{R}^2, \mathbb{R})$. Suppose $\partial_x A + \partial_y B = 0$ for every $(x, y) \in \mathbb{R}^2$. Consider

$$I(x, y) = \int_0^x A(\xi, 0) \mathrm{d}\xi - \int_0^y B(x, \eta) \mathrm{d}\eta.$$

Show that I is a first integral of the dynamical system.

Problem 12: Show that the (local) solution $x(t) \in \mathbb{R}$ of the nonautonomous scalar ODE $\dot{x}(t) = g(t)h(x(t))$ with $x(0) = x_0$ is unique. Assume that g and h are only continuous, with $h(x_0)$ nonzero.

How does this result fit with our example on nonuniqueness of solutions?