Homework Assignments

Dynamical Systems I

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 13: Determine the heteroclinic orbit from $x = -1$ to $x = +1$ for the pendulum

$$x'' + x(1 - x^2) = 0$$

by explicit integration. Use energy and separation of variables.

Problem 14: Consider the closed, sealed-off Müggelsee with predator and prey fish of positive total masses x and y, respectively. Suppose their dynamics obeys the Volterra-Lotka equations

$$\begin{aligned} \dot{x} &= x(\mu - \nu y), \\ \dot{y} &= y(-\varrho + \sigma x), \end{aligned}$$

with positive fixed parameters $\mu, \nu, \varrho, \sigma$. Very ($\varepsilon$-)cautious fishing would change μ to $\tilde{\mu} = \mu - \varepsilon$ and ϱ to $\tilde{\varrho} = \varrho + \varepsilon$, with $\varepsilon > 0$. Why?

Does the time-averaged prey population

$$\overline{x} := \lim_{t \to \infty} \frac{1}{t} \int_0^t x(\tau)\, \mathrm{d}\tau$$

exist? Does $\overline{x}$ increase or decrease, due to fishing? What happens to the total population $\overline{x + y}$?

Hint: Consider time averages of $\dot{x}/x$, $\dot{y}/y$.

Problem 15: Solve the following initial-value problems by separation of variables and determine the maximal time intervals of existence of the solutions:

(i) $\dot{x} = x^2 e^t, \qquad x(0) = 1,$

(ii) $\dot{x} = 1 + x^2, \qquad x(0) = 0,$

(iii) $\dot{x} = 4 - x^2, \qquad x(0) = 0.$

Problem 16: Recall coordinate transformations for vectorfields and flows. Let Φ^t be the flow of the vectorfield $\dot{x} = f(x)$ and let $h : \mathbb{R}^N \to \mathbb{R}^N$ be a diffeomorphism. We use the shorthand $h_* f$ for the vectorfield associated to the transformed ("conjugated") flow $\tilde{\Phi}^t(x) = h(\Phi^t(h^{-1}(x)))$.

(i) Derive a formula for $h_* f$. Compare your result to the formula given in the lecture.

(ii) Prove that for diffeomorphisms $h, \tilde{h}$ we have

$$(h \circ \tilde{h})_* f = h_*(\tilde{h}_* f).$$

(iii) Prove that $(\Phi^t)_* f = f$ for all $t \in \mathbb{R}$.