Homework Assignments

Dynamical Systems I

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`http://dynamics.mi.fu-berlin.de/lectures/`

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Problem 17: Use a numerical integrator to plot the solution $(x(t), y(t))$, $t \in [0, T]$ of the Van-der-Pol oscillator

$$\begin{aligned} \varepsilon \dot{x} &= -y + x(1 - x^2) \\ \dot{y} &= x. \end{aligned}$$

Use the initial condition $x(0) = 1$, $y(0) = 0$ and solve up to time $T = 10$ for values of the parameter $\varepsilon \in \{0.1, 0.05\}$. Use the explicit Euler scheme with stepsizes $h \in \{0.075, 0.03\}$ for $\varepsilon = 0.1$ and with stepsizes $h \in \{0.03, 0.006\}$ for $\varepsilon = 0.05$. Discuss the results. You must label your plots (handwritten is OK).

We recommend using either `matlab` or `octave` for this exercise; both are available on the university computers and `octave` is actually free. You find two program files on the homepage: `expliciteuler.m` implements the explicit euler scheme and `lotka_volterra_example.m` creates some plots for Exercise 14. You may use these examples as a starting point and modify or use them.

Problem 18: [Duffing equation] Consider the potential $V(x) = x^4 - 2x^2 + 1$ with the associated pendulum equation $\ddot{x} = -V'(x)$.

Prove or disprove: For every $p > 0$, there exists a perdiodic orbit, which is symmetric to the origin and has minimal period p.

Problem 19: Consider (point-sized) Jerry runnning along the x-axis in the plane with speed 1, starting at the origin. At the same time, (point-sized) Tom starts at $(x, y) = (0, 1)$. Tom always runs towards Jerry, directly and with the same speed 1.

Does Tom ever catch Jerry?

Hint: Use polar coordinates for their and solve the resulting system, e.g. by separation of variables.

Problem 20: Consider the pendulum equation

$$\ddot{x} + g(x) = 0$$

for a continuous, odd function $g : \mathbb{R} \to \mathbb{R}$, i.e. $g(-x) = -g(x)$ for all $x \in \mathbb{R}$. Assume $g(x) \cdot x > 0$ for all $x \neq 0$. Let $p(g, a) > 0$ be the minimal period of the solution to the initial value $x(0) = a > 0$, $\dot{x}(0) = 0$.

Prove:

(i) If $g_1(x) < g_2(x)$ for all $x > 0$ then $p(g_1, a) > p(g_2, a)$ for all $a > 0$.

(ii) [Soft spring] If $x \mapsto g(x)/x$ is strictly monotonically decreasing for $x > 0$, then $a \mapsto p(g, a)$ is strictly monotonically increasing for $a > 0$.

Hint: $y(t) := \frac{a_1}{a_2} x(t)$ solves the equation $\ddot{y} + \tilde{g}(y) = 0$ with $\tilde{g}(y) := \frac{a_1}{a_2} g(\frac{a_2}{a_1} y)$.