Homework Assignments

Dynamical Systems I

Bernold Fiedler, Bernhard Brehm

http://dynamics.mi.fu-berlin.de/lectures/

due date: Friday, May 30, 2014

Problem 21: Consider the synthesis of ammonia from (expensive) hydrogen H_2 and (cheap as air) nitrogen N_2:

$$N_2 + 3H_2 \leftrightarrows 2NH_3$$

Denote the reaction rates by $k_{\rightarrow}$, $k_{\leftarrow}$ and determine the kinetic equations for the vector $x = (x_{N_2}, x_{H_2}, x_{NH_3})$ of non-negative concentrations. Determine the ω-limit set of initial data with $x_{NH_3} = 0$.

Extra credit: Which choice of initial data yields the highest gain NH_3/H_2?

Problem 22: Consider the initial-value problem

$$\dot{x} = f(x) := x, \qquad x(0) = x_0 := 1$$

on the time interval $0 \leq t \leq T$ for fixed $T > 0$. Calculate an approximate solution $x(T)$ analytically

(i) by Picard iteration, i.e. determine the n-th Picard iterate $x^{[n]}(T)$;

$$x^{[k+1]}(t) := x_0 + \int_0^t f(x^{[k]}(\tau))\,\mathrm{d}\tau, \qquad x^{[0]}(t) \equiv x_0$$

(ii) by implicit Euler scheme, i.e. determine the value x_n after n Euler steps of stepsize $h = T/n$,

$$x_{k+1} = x_k + hf(x_{k+1}), \qquad x_0 = x_0$$

Compare $x^{[n]}(T)$ with x_n and the explicit $x(T)$!

Problem 23: Let $f : \mathbb{R}^N \to \mathbb{R}^N$ be locally Lipschitz continuous. Let $I(x_0)$ denote the maximal time of existence for the solution $x(t)$ to the initial value problem

$$\dot{x}(t) = f(x(t)) \qquad x(0) = x_0.$$

Show that $I(x(\tau)) = I(x_0) - \tau$ for all $\tau \in I(x_0)$.

Problem 24: The following is an integrated Gronwall-Lemma:

Let $\alpha, \beta, u : [0, T] \to \mathbb{R}_+$ be continuous functions. Suppose that the following inequality holds:

$$u(t) \leq \alpha(t) + \int_0^t \beta(s)u(s)\mathrm{d}s \qquad \forall t \in [0, T].$$

Then $u(t) \leq \alpha(t) + \int_0^t B^{t,s}\beta(s)\alpha(s)\mathrm{d}s$, where $B^{t,s} = \exp\left(\int_s^t \beta(r)\mathrm{d}r\right)$.

Prove the integrated Gronwall-Lemma for the simplified case where $u, \alpha \in C^1$. You may chose a proof which is similiar to the one in the lecture, but you may not cite the theorem from the lecture.

Extra credits: Prove the the integrated Gronwall-Lemma for $u, \alpha \in C^0$.