Homework Assignments

Dynamical Systems I

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 29: Consider the Banach space BC^1 of continuously differentiable vector fields $f : X \to X = \mathbb{R}^N$ with $\|f\|_{BC^1} := \sup_{x\in X}(|f(x)| + |f'(x)|) < \infty$. Let f, g be vector fields in BC^1 and let $x(f,t)$ denote the solution at time t of the differential equation $\dot{x}(t) = f(x(t)),\ x(0) = x_0$. Is the map

$$x(t,\cdot) :\ BC^1 \to X, \qquad f \mapsto x(t,f),$$

differentiable with respect to $f \in BC^1$, for fixed t ? If so, then which differential equation is solved by the Gateaux-derivative $v(t) := \mathrm{D}_f x(t,f)g$?

Problem 30: Determine the maximal interval of existence $t \in (t_-(z_0), t_+(z_0))$ for any complex solution $z = z(t)$ of $\dot{z} = z^2$ with initial condition $z(0) = z_0$. At which z_0 is the dependence of $t_\pm(z_0)$ on z_0

(i) upper-semicontinuous ,

(ii) lower-semicontinuous ,

(iii) continuous ?

Problem 31: Calculate the solutions of the following linear differential equations

(i) $$\dot{x} = \begin{pmatrix} -3 & 0 & 2 \\ -1 & -2 & 5 \\ -1 & 0 & 0 \end{pmatrix} x, \qquad x(0) = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

(ii) $$\dot{x} = \begin{pmatrix} 3 & 0 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & -2 \end{pmatrix} x, \qquad x(0) = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}.$$

Problem 32: Can real $n \times n$-matrices A, B fail to commute if $e^A = e^B = e^{A+B} = \mathrm{id}$? Prove your claim!