Homework Assignments

Dynamical Systems I

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Problem 33: Let $A = (a_{ij})_{1 \leq i,j \leq n}$ be a real $n \times n$-matrix. Prove: The coefficients of the matrix $\exp(At)$ are non-negative for all $t \geq 0$ if, and only if, $a_{ij} \geq 0$ for all $i \neq j$.

Hint: It suffices to consider the case $a_{ij} \geq 0$ for *all* i, j. (Why?)

Problem 34: We want to understand the damped linear pendulum

$$\ddot{x} + 2\,\nu\dot{x} + \omega_0^2 x = 0$$

with parameters $\nu, \omega_0 > 0$ and initial conditions $x(0) = 0$, $\dot{x}(0) = 1$.

(i) Determine the explicit solution, for all ν, ω_0. Sketch (yes, by hand!) phase portraits and a diagram of the (ν, ω_0)-plane indicating different qualitative behavior.

(ii) How does the phase portrait change at the boundary between different zones in the (ν, ω_0)-plane, for instance due to a change of the damping ν? How do you reconcile the discontinuities in the phase portraits of the Jordan normal form with differentiable dependence of the flow on the parameters ν, ω_0?

Problem 35: How many decimal digits does the 1.000.000.000.001-st Fibonacci number have, i.e. $x_n = x_{n-1} + x_{n-2}$ with $x_0 = 0$ and $x_1 = 1$?

Problem 36: [LISSAJOUS figures] Let A be a real, symmetric, positive definite 2×2-matrix. Consider the Hamilton function $H(x, \dot{x}) = \frac{1}{2}(\dot{x}^{\mathrm{T}}\dot{x} + x^{\mathrm{T}}Ax)$ and the associated Hamiltonian system

$$(*) \qquad \ddot{x} + Ax = 0.$$

(i) Transform $(*)$ into a system of decoupled pendulum equations (ω_1, ω_2 real):

$$(**) \qquad \begin{cases} \ddot{y}_1 + \omega_1^2 y_1 &= 0, \\ \ddot{y}_2 + \omega_2^2 y_2 &= 0, \end{cases}$$

(ii) Sketch the solution $(x_1(t),\ x_2(t))$ of $(*)$ for

$$A = \begin{pmatrix} 5 & -4 \\ -4 & 5 \end{pmatrix}$$

with initial conditions $x_1 = x_2 = 1\,;\ \dot{x}_1 = \dot{x}_2 = 0.$

(iii) Try to describe the solutions for other ratios ω_1/ω_2, in words.